

**Answer to Essential Question 6.4:** In each case, the force of friction (static friction if there is no slipping) between the man's shoes and the canoe causes the changes in the canoe's motion.

### 6-5 Playing with a Constant Acceleration Equation

Once again, let's start with a familiar relationship and look at in a new way to come up with a powerful idea. Return to one of our constant acceleration equations:  $v_x^2 = v_{ix}^2 + 2\bar{a}_x\Delta\bar{x}$ . If we

re-arrange this equation to solve for the acceleration, we get:  $\bar{a}_x = \frac{v_x^2 - v_{ix}^2}{2\Delta\bar{x}}$ .

Substituting this into Newton's second law,  $\bar{F}_{net,x} = m\bar{a}_x$ , gives, after some re-arranging:

$$\frac{1}{2}mv_x^2 - \frac{1}{2}mv_{ix}^2 = \bar{F}_{net,x}\Delta\bar{x}.$$

We can do the same thing in the  $y$ -direction. Adding the  $x$  and  $y$  equations gives:

$$\left(\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2\right) - \left(\frac{1}{2}mv_{ix}^2 + \frac{1}{2}mv_{iy}^2\right) = \bar{F}_{net,x}\Delta\bar{x} + \bar{F}_{net,y}\Delta\bar{y}.$$

Recognizing that  $v_x^2 + v_y^2 = v^2$ , the left side of the equation can be simplified:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 = \bar{F}_{net,x}\Delta\bar{x} + \bar{F}_{net,y}\Delta\bar{y}.$$

The right side can also be simplified, because its form matches a dot product:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 = \bar{F}_{net} \bullet \Delta\bar{r} = F_{net}\Delta r \cos\theta, \quad (\text{Equation 6.5})$$

where  $\theta$  is the angle between the net force  $\bar{F}_{net}$  and the displacement  $\Delta\bar{r}$ .

So, we've now come up with two more useful concepts, which we name and define here.

**Kinetic energy** is energy associated with motion:

$$K = \frac{1}{2}mv^2 \quad ; \quad (\text{Equation 6.6: Kinetic energy})$$

**Work** relates force and displacement  $W = \bar{F}_{net} \bullet \Delta\bar{r} = F_{net}\Delta r \cos\theta$ . (Eq. 6.7: Work)

Both work and kinetic energy have units of joules (J), and they are both scalars.

Equation 6.5, when written in the form below, is known as the **work-kinetic energy theorem**. In this case, the work is the work done by the net force.

$$\Delta K = W_{net} = F_{net}\Delta r \cos\theta, \quad (\text{Eq. 6.8: The work-kinetic energy theorem})$$

where  $\theta$  is the angle between the net force  $\bar{F}_{net}$  and the displacement  $\Delta\bar{r}$ .

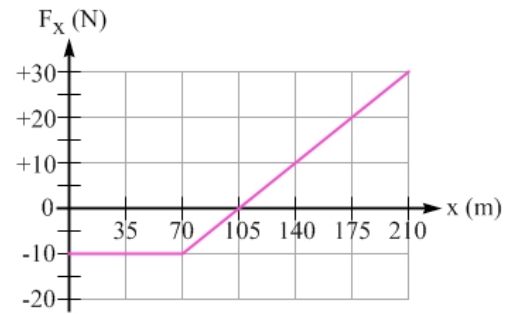
In general, when a force is perpendicular to the displacement, the force does no work. If the force has a component parallel to the displacement, the force does positive work. If the force has a component in the direction opposite to the displacement, the force does negative work.

Compare Exploration 6.5 to Exploration 6.2, in which Suki was riding her bike.

### EXPLORATION 6.5 – A hard-working cyclist

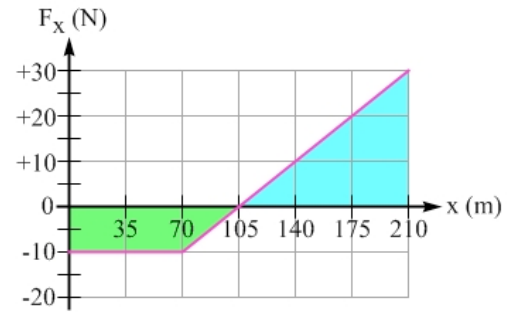
Peter is riding his bicycle in a straight line on a flat road. Peter and his bike have a total mass of 60 kg and, at  $t = 0$ , he is traveling at 8.0 m/s. For the first 70 meters, he coasts. When Peter realizes he is slowing down, he pedals so that the static friction force exerted on the bike by the road increases linearly with distance from 0 to 40 N, in the direction Peter is traveling, over the next 140 meters. A constant 10 N resistive force acts on Peter and the bicycle the entire time.

**Step 1 - Sketch a graph of the net force acting on Peter and his bicycle as a function of position.** Take the positive direction to be the direction Peter is traveling. In the vertical direction, the normal force balances the force of gravity, so we can focus on the horizontal forces. For the first 70 m, we have only the 10 N resistive force, which opposes the motion and is thus in the negative direction. For the next 140 m, we have to account for the friction force, which acts in the direction of motion, and the resistive force. We can account for their combined effect by drawing a straight line, as in Figure 6.12, that goes from  $-10$  N at  $x = 70$  m to  $+30$  N ( $40$  N  $-$   $10$  N) at  $x = 210$  m.



**Figure 6.12:** A graph of the net force acting on Peter and his bicycle, as a function of position.

**Step 2 - What is Peter's speed at  $x = 210$  m?** Let's use the area under the  $F_{net}$  versus position graph, between  $x = 0$  and  $x = 210$  m, to find the net work over that distance. This area is shown in Figure 6.13, split into a negative area for the region  $x = 0$  to  $x = 105$  m, and a positive area between  $x = 105$  m and  $x = 210$  m. Each box on the graph has an area of  $10 \text{ N} \times 35 \text{ m} = 350 \text{ N m}$ . The negative area covers two-and-a-half boxes on the graph, while the positive area covers four-and-a-half boxes, for a net positive area of 2 boxes, or  $700 \text{ N m}$ .



**Figure 6.13:** The area within the shaded regions represents the area under the curve for the region from  $x = 0$  to  $x = 210$  m.

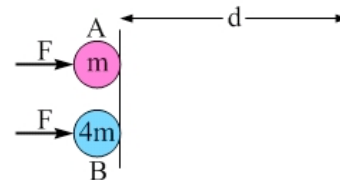
The net area under the curve in Figure 6.13 is the net work done on Peter and the bicycle, which is the change in kinetic energy ( $W_{net} = \Delta K = K_f - K_i$ ). Thus, the final kinetic energy is:

$$K_f = K_i + W_{net} = \frac{1}{2}mv_i^2 + W_{net} = \frac{1}{2}(60 \text{ kg})(8.0 \text{ m/s})^2 + 700 \text{ N m} = 1920 \text{ J} + 700 \text{ J} = 2620 \text{ J}.$$

$$\text{Solving for the final speed from } K_f = (1/2)mv_f^2 \text{ gives: } v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2 \times 2620 \text{ J}}{60 \text{ kg}}} = 9.3 \text{ m/s}.$$

**Key idea:** The area under the net force-versus-position graph for a particular region is the work, and the change in kinetic energy, over that region. **Related End-of-Chapter Exercises: 48, 49.**

**Essential Question 6.5:** Initially, objects  $A$  and  $B$  are at rest.  $B$ 's mass is four times larger than  $A$ 's mass. Identical net forces are applied to the objects, as shown in Figure 6.14. Each force is removed once the object it is applied to has accelerated through a distance  $d$ . After the forces are removed, which object has more (a) kinetic energy? (b) momentum?



**Figure 6.14:** An overhead view of two objects,  $A$  and  $B$ , experiencing the same net force  $F$  as they move from rest through a distance  $d$ .