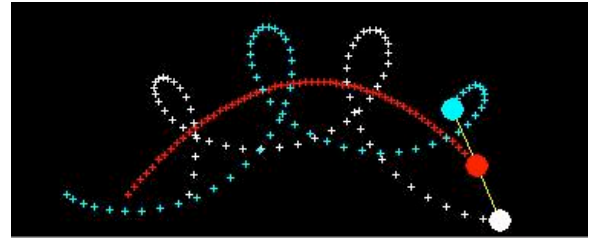


**Answer to Essential Question 6.3:** No matter how the masses of the two carts in Exploration 6.3B compare, the center of mass of the system remains at rest at the balance point of the track.

## 6-4 Center of Mass

In the previous chapters, we treated everything as a particle, or, equivalently, as a ball. A ball, or particle, thrown through the air follows a parabolic path. What if you take a non-spherical object (a pen, for instance) and throw it so it spins? Most points on the object follow complicated paths, but the center of mass still follows a parabolic curve, as shown in Figure 6.9.



**Figure 6.9:** The motion of three balls on a stick. Only the red ball, located at the center of mass of the system, follows the parabolic path characteristic of free fall.

For a uniform object, the center of mass is located at the geometric center of the object. In general, the center of mass of an object, or a collection of objects, is given by Equation 6.4.

The **center of mass** is the point on an object that moves as though all the mass of the object is concentrated there. The  $x$ -coordinate of the center of mass is given by:

$$X_{CM} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \dots}{m_1 + m_2 + m_3 + \dots} \quad (\text{Equation 6.4: Position of the center of mass})$$

where the  $m$ 's represent the masses of different objects in the system (or of various pieces of a single object) and the  $x$ 's represent the  $x$ -coordinates of those objects or pieces. Similar equations give us the  $y$  and  $z$ -coordinates of the center of mass.

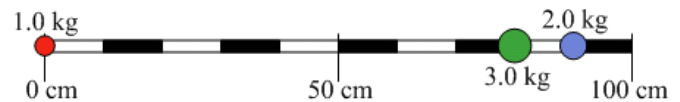
### EXAMPLE 6.4A – Three balls on a stick

Three balls are placed on a meter stick. Ball 1, at the 0-cm mark, has a mass of 1.0 kg. Ball 2, at the 80-cm mark, has a mass of 3.0 kg. Ball 3, at the 90-cm mark, has a mass of 2.0 kg.

- If the meter stick has negligible mass, where is the system's center of mass?
- If the meter stick has a mass of 2.0 kg, where is the system's center of mass?

#### SOLUTION

(a) As usual, let's begin with a diagram of the situation (see Figure 6.10).



**Figure 6.10:** A diagram showing the position of the three balls on the meter stick.

To find the center of mass, we can substitute the given values into Equation 6.4:

$$X_{CM} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_1 + m_2 + m_3} = \frac{0 \times 1.0 \text{ kg} + (80 \text{ cm})(3.0 \text{ kg}) + (90 \text{ cm})(2.0 \text{ kg})}{1.0 \text{ kg} + 2.0 \text{ kg} + 3.0 \text{ kg}} = \frac{420 \text{ kg cm}}{6.0 \text{ kg}} = 70 \text{ cm}.$$

(b) If the stick's mass is uniformly distributed, we can treat the stick as a fourth ball, with a mass of 2.0 kg, located at the 50-cm mark. Making use of the result from part (a), which says that the first three balls are equivalent to a single 6.0-kg ball located at the 70-cm mark, we get:

$$X'_{CM} = \frac{(70 \text{ cm})(6.0 \text{ kg}) + (50 \text{ cm})(2.0 \text{ kg})}{6.0 \text{ kg} + 2.0 \text{ kg}} = \frac{520 \text{ kg cm}}{8.0 \text{ kg}} = 65 \text{ cm}.$$

**Related End-of-Chapter Exercises: 31, 32.**

The center of mass is particularly useful in systems experiencing no net external force. In such systems, the motion of the system's center of mass is unchanged, even if the motion of different parts of the system changes. This is a consequence of Newton's Second Law. Without a net external force acting, the acceleration of the center of mass of the system is zero. Different parts of the system can accelerate, but the forces associated with these accelerations cancel because the net force on the system is zero. Let's now consider an example of such a system.

**EXAMPLE 6.4B – Canoe move the center of mass?**

A man, with a mass of 90 kg, stands 2.0 m from the center of a 30 kg canoe that is floating on the calm water of a lake. Both the man and the canoe are initially at rest.

(a) If the man then moves to the point 2.0 m on the opposite side of the center of the canoe from where he starts, how far does the canoe move?

(b) How far does the man actually move relative to a fixed point on the shore?

**SOLUTION**

(a) We could solve this problem formally, but let's solve it conceptually by looking at Before and After pictures in Figure 6.11. First, let's determine the position of the center of mass of the system in the Before picture, before the man changes position.

Define the man's initial position as the origin (you can pick a different origin if you want), and assume the canoe's center of mass to be the middle of the canoe. We get:

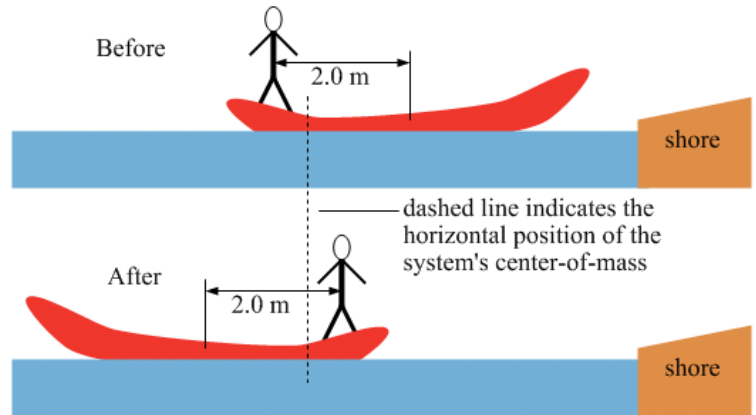
$$X_{CM} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{0 \times 90 \text{ kg} + (2.0 \text{ m})(30 \text{ kg})}{90 \text{ kg} + 30 \text{ kg}} = \frac{60 \text{ kg} \cdot \text{m}}{120 \text{ kg}} = 0.50 \text{ m}$$

In the Before picture, the man is 50 cm to the left, and the canoe's center of mass is 1.5 m to the right, of the system's center of mass. The canoe's center of mass is three times farther from the system's center of mass than the man is because the canoe's mass is 1/3 of the man's mass. Because no net external force acts on the canoe-man system, when the man moves to the right, the canoe moves left in such a way that the system's center of mass remains at rest. The man moves to a position that is a mirror image of his initial position, so the After picture is a mirror image of the Before picture (placing the mirror at the system's center of mass). The canoe's center of mass moves from 1.5 m to the right of the system's center of mass to 1.5 m to the left of the system's center of mass, for a net displacement of 3.0 m to the left.

(b) Applying a similar analysis to the man, the man moves from 0.5 m to the left of the system's center of mass to 0.5 m to the right, a net displacement of 1.0 m right. Equivalently, the man moves 4.0 m to the right relative to the canoe while the canoe moves 3.0 m to the left with respect to the shore, so the man ends up moving just 1.0 m right relative to the shore.

**Related End-of-Chapter Exercise: 33.**

**Essential Question 6.4:** In Example 6.4B, what force makes the canoe move when the man starts to move? What force stops the canoe when the man stops?



**Figure 6.11:** The position of the man and the canoe before and after the man moves from one end of the canoe to the other.