***Answer to Essential Question 6.1*:** The force is constant, and Equation 6.3 tells us that the velocity increases linearly with time. Thus, at a time $2T$, the velocity will be $2v$ directed down.

6-2 Relating Momentum and Impulse

In this section, we will apply the general method from the end of Section 6-1 to solve a problem using the concepts of impulse and momentum.

EXPLORATION 6.2 – An impulsive bike ride

Suki is riding her bicycle, in a straight line, along a flat road. Suki and her bike have a combined mass of 50 kg. At $t = 0$, Suki is traveling at 8.0 m/s. Suki coasts for 10 seconds, but when she realizes she is slowing down, she pedals for the next 20 seconds. Suki pedals so that the static friction force exerted on the bike by the road increases linearly with time from 0 to 40 N, in the direction Suki is traveling, over that 20-second period. Assume there is constant 10 N resistive force, from air resistance and other factors, acting on her and the bicycle the entire time.

Step 1 - *Sketch a diagram of the situation.* The diagram is shown in Figure 6.2, along with the free-body diagram that applies for the first 10 s and the free-body diagram that applies for the 20-second period while Suki is pedaling.

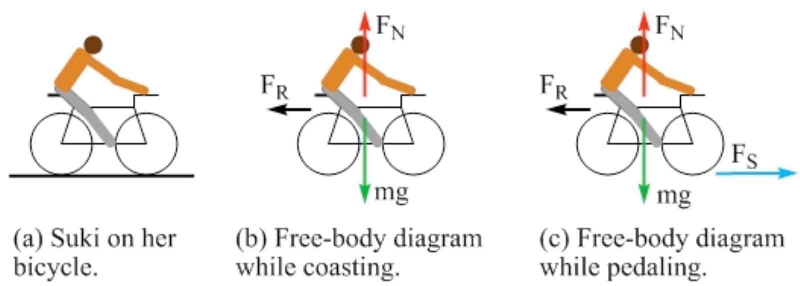

Figure 6.2: A diagram of (a) Suki on her bike, as well as free-body diagrams while she is (b) coasting and while she is (c) pedaling. Note that in free-body diagram (c), the static friction force $\vec{F}_S$ gradually increases because of the way Suki pedals.

Step 2 - *Sketch a graph of the net force acting on Suki and her bicycle as a function of time.* Take the positive direction to be the direction Suki is traveling. In the vertical direction, the normal force exactly balances the force of gravity, so we can focus on the horizontal forces. For the first 10 seconds, we have only the 10 N resistive force, which acts to oppose the motion and is thus in the negative direction. For the next 20 seconds, we have to account for the friction force that acts in the direction of motion and the resistive force. We can account for their combined effect by drawing a straight line that goes from –10 N at $t = 10$ s, to +30 N (40 N – 10N) at $t = 30$ s. The result is shown in Figure 6.3.

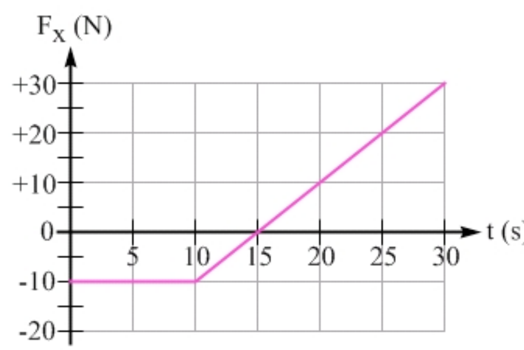

Figure 6.3: A graph of the net force acting on Suki and her bicycle as a function of time.

Step 3 - *What is Suki's speed at t = 10 s?* Let's apply Equation 6.3, which we can write as:

$$\vec{F}_{net}\,\Delta t = \Delta(m\vec{v}) = m\,\Delta\vec{v} = m\left(\vec{v}_{10s} - \vec{v}_i\right).$$

Solving for the velocity at $t = 10$ s gives:

$$\vec{v}_{10s} = \vec{v}_i + \frac{\vec{F}_{net}\,\Delta t}{m} = +8.0\text{ m/s} + \frac{(-10\text{ N})(10\text{ s})}{50\text{ kg}} = +8.0\text{ m/s} - 2.0\text{ m/s} = +6.0\text{ m/s}.$$

Thus, Suki's speed at t = 10 s is 6.0 m/s. We can also obtain this result from the force-versus-time graph, by recognizing that the impulse, $\vec{F}_{net}\,\Delta t$, represents the area under this graph over some time interval Δt. Let's find the area under the graph, over the first 10 seconds, shown in Figure 6.4. The area is negative, because the net force is negative over that time interval. The area under the graph is the impulse:

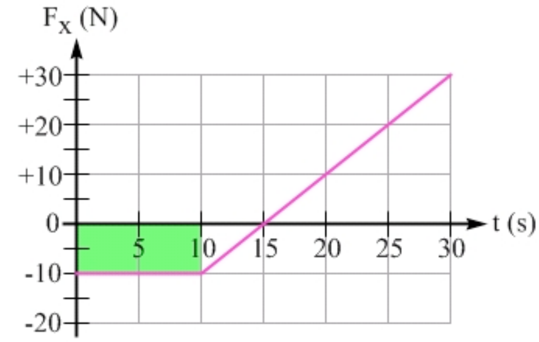

Figure 6.4: The rectangle represents the area under the graph for the first 10 s. The area is negative, because the force is negative.

$$\vec{F}_{net}\,\Delta t = -10\text{ N}\times 10\text{ s} = -100\text{ N s} = -100\text{ kg m/s}.$$

From Equation 6.3, we know the impulse is equal to the change in momentum. Suki's initial momentum is $m\vec{v}_i = 50\text{ kg}\times(+8.0\text{ m/s}) = +400\text{ kg m/s}$. Her momentum at t = 10 s is therefore $+400\text{ kg m/s} - 100\text{ kg m/s} = +300\text{ kg m/s}$. Dividing this by the mass to find the velocity at t = 10 s confirms what we found above:

$$\vec{v}_{10s} = \frac{\vec{p}_{10s}}{m} = \frac{\vec{p}_i + \Delta\vec{p}}{m} = \frac{+400\text{ kg m/s} - 100\text{ kg m/s}}{50\text{ kg}} = \frac{+300\text{ kg m/s}}{50\text{ kg}} = +6.0\text{ m/s}.$$

Step 4 - *What is Suki's speed at t = 30 s?* Let's use the area under the force-versus-time graph, between t = 10 s and t = 30 s, to find Suki's change in momentum over that 20-second period. This area is highlighted in Figure 6.5, split into a negative area for the time between t = 10 s and t = 15 s, and a positive area between t = 15 s and t = 30 s. These regions are triangles, so we can use the equation for the area of a triangle, $0.5\times base\times height$. The area under the curve, between 10 s and 15 s, is $0.5\times(5.0\text{ s})\times(-10\text{ N}) = -25\text{ kg m/s}$. The area between 15 s and 30 s is $0.5\times(15\text{ s})\times(30\text{ N}) = +225\text{ kg m/s}$. The total area (total change in momentum) is +200 kg m/s.

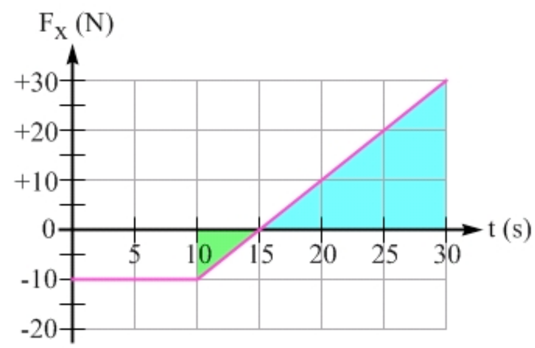

Figure 6.5: The shaded regions correspond to the area under the curve for the time interval from t = 10 s to t = 30 s.

Note that another approach is to multiply the average net force acting on Suki and the bicycle (+10 N) over this interval, by the time interval (20 s), for a +200 kg m/s change in momentum.

In step 3, we determined that Suki's momentum at t = 10 s is +300 kg m/s. With the additional 200 kg m/s, the net momentum at t = 10 s is +500 kg m/s. Dividing by the 50 kg mass gives a velocity at t = 30 s of +10 m/s.

Key idea for the graphical interpretation of impulse: The area under the net force versus time graph for a particular time interval is equal to the change in momentum during that time interval. **Related End-of-Chapter Exercises: 24, 27 – 30.**

Essential Question 6.2: Return to the 30-second interval covered in Exploration 6.2. At what time during this period does Suki reach her minimum speed?