6-1 Rewriting Newton's Second Law

In this chapter, we will begin by taking a look at two ideas that we are familiar with from previous chapters. Let's see what happens when we combine Newton's second law, $\vec{F}_{net} = m\vec{a}$, with the definition of acceleration, $\vec{a} = \Delta\vec{v}/\Delta t$. This gives $\vec{F}_{net} = m\Delta\vec{v}/\Delta t$. Let's be a bit creative and write this relationship in a form more as Newton himself originally did:

$$\vec{F}_{net} = \frac{\Delta(m\vec{v})}{\Delta t}$$ (Equation 6.1: **General form of Newton's Second Law**)

Equation 6.1 is more general than Newton's second law stated in the form $\vec{F}_{net} = m\vec{a}$, because equation 6.1 allows us to work with systems (such as rockets) where the mass changes. $\vec{F}_{net} = m\vec{a}$ applies only to systems where the mass is constant, although so many systems have constant mass that we find this form of the equation to be very useful.

The general form of Newton's second law connects the net force on an object with the rate of change of the quantity $m\vec{v}$. This quantity has a name, which you may already be familiar with.

> An object's **momentum** is the product of its mass and its velocity. Momentum is a vector, pointing in the direction of the velocity. The symbol we use to represent momentum is $\vec{p}$.
>
> $$\vec{p} = m\vec{v}.$$ (Equation 6.2: **Momentum**)

Equation 6.1 can be re-arranged to read: $\vec{F}_{net}\,\Delta t = \Delta(m\,\vec{v}) = \Delta\vec{p}$.

Thus, to change an object's momentum, all we have to do is to apply a net force for a particular time interval. To produce a larger change in momentum, we can apply a larger net force or apply the same net force over a longer time interval. The product of the net force and the time interval over which the force is applied is such an important quantity that we should name it, too.

> The product of the net force and the time interval over which the force is applied is called **impulse**. An impulse produces a change in momentum. An impulse is a vector.
>
> $$\vec{F}_{net}\,\Delta t = \Delta(m\,\vec{v}) = \Delta\vec{p}.$$ (Equation 6.3: **Impulse**)

EXPLORATION 6.1 – Hitting the boards
Just before hitting the boards of an ice rink, a hockey puck is sliding along the ice at a constant velocity. The components of the velocity are 3.0 m/s in the direction perpendicular to the boards and 4.0 m/s parallel to the boards. After bouncing off the boards, the puck's velocity component perpendicular to the boards is 2.0 m/s and the component parallel to the boards is unchanged. The puck's mass is 160 g.

Step 1 - *What is the impulse applied to the puck by the boards?* Let's sketch a diagram (see Figure 6.1) to help visualize what is going on. Impulse is the product of the net force and the time--but we don't know the net force so we can't get impulse that way. Impulse is also equal to the change in momentum, as we can see from equation 6.3, so let's figure out that change.

Remember that momentum, impulse, and velocity are all vectors. Let's choose a coordinate system with the positive *x*-direction to the right and the positive *y*-direction up. It is important to notice that there has been no change in the puck's velocity in the *y*-direction, so there is no change in momentum, no impulse, and no net force in that direction. We can focus on the *x*-direction to answer the question.

The puck's mass is constant, so the puck's momentum changes because there is a change in velocity. What is the magnitude of the change in the puck's velocity in the *x*-direction, perpendicular to the boards? It's tempting to say 1.0 m/s, but it is actually 5.0 m/s. That result comes from:

$$\Delta \vec{v}_x = \vec{v}_x - \vec{v}_{ix} = +2.0 \text{ m/s} - (-3.0 \text{ m/s}) = +5.0 \text{ m/s} .$$

Figure 6.1: The puck's velocity components before and after it collides with the boards.

Knowing the change in velocity, we can find the impulse, which is in the +*x*-direction:

$$\Delta \vec{p}_x = m\,\Delta \vec{v}_x = 0.16 \text{ kg} \times (+5.0 \text{ m/s}) = +0.80 \text{ kg m/s} .$$

Step 2 - *If the puck is in contact with the boards for 0.050 s, what is the average force applied to the puck by the boards?* The force varies over the 0.050 s the puck is in contact with the boards, but we can get the average force from:

$$\vec{F}_x = \frac{\Delta \vec{p}_x}{\Delta t} = \frac{+0.80 \text{ kg m/s}}{0.050 \text{ s}} = +16 \text{ N}$$
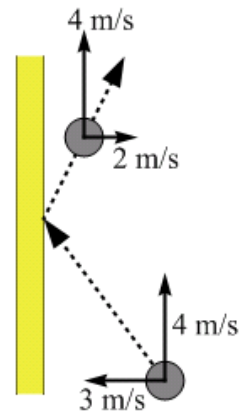

Key ideas for impulse and momentum: Analyzing situations from an impulse-momentum perspective can be very useful, as it allows us to directly connect force, velocity, and time. It is absolutely critical to account for the fact that momentum, impulse, force, and velocity are all vectors when carrying out such an analysis.   **Related End-of-Chapter Exercises: 14, 15**

Let's now summarize a general method we can use to solve a problem involving impulse and momentum. We will apply this method in Exploration 6.2.

Solving a Problem Involving Impulse and Momentum
A typical impulse-and-momentum problem relates the net force acting on an object over a time interval to the object's change in momentum. A method for solving such a problem is:

1. Draw a diagram of the situation.
2. Add a coordinate system to the diagram, showing the positive direction(s). Keeping track of direction is important because force and momentum are vector quantities.
3. Organize what you know, and what you're looking for, such as by drawing one or more free-body diagrams, or drawing a graph of the net force as a function of time.
4. Apply equation 6.3 $\left(\vec{F}_{net}\Delta t = \Delta(m\vec{v}) \right)$ to solve the problem.

Essential Question 6.1: At some time *T* after a ball is released from rest, the force of gravity has accelerated that ball to a velocity *v* directed straight down. Taking into account impulse and momentum, what is the ball's velocity at a time 2*T* after being released?