Answer to Essential Question 5.5: The faster the ride goes, the larger the normal force that the wall exerts on the rider. The larger the normal force is, the larger the maximum possible force of static friction that the wall can exert on the rider. The upward static friction force must balance the downward force of gravity. To provide a margin of safety, the maximum possible force of static friction should exceed the rider's weight by a significant amount. If the ride is too slow, the normal force is reduced, reducing the maximum possible force of static friction. If the maximum possible force of static friction drops below the rider's weight, the rider will slide down the wall. A similar situation could occur if the coefficient of static friction, associated with the interaction between the wall and the rider's clothes, is too small.

5-6 Solving Problems Involving Uniform Circular Motion

Let's investigate a typical circular-motion situation in some detail, although first we should slightly modify our general approach to solving problems using forces. Usually, the method that we follow in a uniform circular motion situation is identical to the approach that we use for other problems involving Newton's Second Law, where we apply the equation $\sum \vec{F} = m\vec{a}$. However, for uniform circular motion, the acceleration has the special form of Equation 5.3, $a_c = v^2 / r$. Thus, when we apply Newton's Second Law, it has a special form.

> The special form of Newton's Second Law for uniform circular motion is:
>
> $$\sum \vec{F} = \frac{mv^2}{r}$$ (Eq. 5.4: **Newton's Second Law for uniform circular motion**)
>
> where the net force, and the acceleration, is directed toward the center of the circle.

EXAMPLE 5.6 – A ball on a string

You are whirling a ball of mass *m* in a horizontal circle at the end of a string of length *L*. The ball has a constant speed *v*, and the string makes an angle *θ* with the vertical.

(a) What is the tension in the string? Express your answer in terms of *m*, *g*, and *θ*.
(b) What is *v*? Express your answer in terms of *m*, *L*, *g*, and/or *θ*.

SOLUTION

Let's apply the general method for solving problems using Newton's Laws. The first step is to draw a diagram (see Figure 5.16a) showing the ball, the string, and the circular path followed by the ball. The next step is to draw a free-body diagram showing the forces acting on the ball. Although the ball is going in a horizontal circle, the string is at an angle. As shown in Figure 5.16b, only two forces act on the ball, the downward force of gravity and the force of tension that is directed away from the ball along the string.

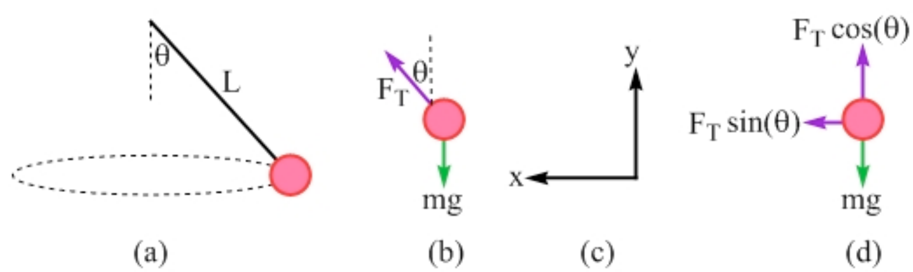

Figure 5.16: (a) A diagram and (b) free-body diagram for a ball being whirled in a horizontal circle at the end of a string. (c) An appropriate coordinate system. (d) A free-body diagram, with force components aligned with the coordinate system.

Now, choose an appropriate coordinate system. The key is to align the coordinate system with the acceleration. Because the ball is experiencing uniform circular motion, the acceleration is directed horizontally toward the center of the circle. We can choose a coordinate system with axes that are horizontal and vertical, as in Figure 5.16c. Finally, split the tension into components, with $F_T \cos\theta$ vertically up and $F_T \sin\theta$ toward the center of the circle, as in Figure 5.16d.

(a) To find an appropriate expression for the tension, we can apply Newton's second law in the *y*-direction. Because there is no acceleration vertically, we have:

$$\sum \vec{F}_y = m\vec{a}_y = 0 .$$

Looking at the free-body diagram to evaluate the left-hand side of this equation gives:

$$+F_T \cos\theta - mg = 0 .$$

Solving for the tension gives: $F_T = \dfrac{mg}{\cos\theta}$.

(b) To find an expression for the speed of the ball, let's apply Newton's second law in the *x*-direction. The positive *x*-direction is toward the center of the circle, in the direction of the centripetal acceleration, so we apply the special form of Newton's second law that is appropriate for use in circular motion situations. The general equation is:

$\sum \vec{F}_x = \dfrac{mv^2}{r}$, where the acceleration is directed toward the center of the circle.

Looking at the free-body diagram in Figure 5.16d, we see that there is only one force in the *x*-direction, so:

$$+F_T \sin\theta = \frac{mv^2}{r} .$$

A common error in this situation is to assume that *r*, the radius of the circular path, is equal to *L*, the length of the string. Referring to Figure 5.17, however, it can be seen that $r = L\sin\theta$.

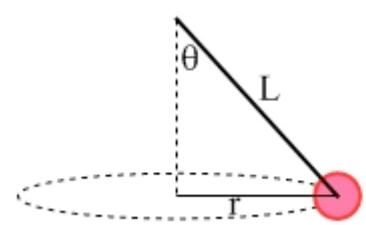

Figure 5.17: Note that *r*, the radius of the circular path, is not the same as *L*, the length of the string.

Substituting that into our equation gives:

$$F_T \sin\theta = \frac{mv^2}{L\sin\theta}, \qquad \text{so} \qquad v^2 = \frac{F_T L \sin^2\theta}{m} .$$

Using our result from part (a) to eliminate F_T gives:

$$v^2 = \frac{mgL\sin^2\theta}{m\cos\theta} = \frac{gL\sin^2\theta}{\cos\theta} .$$

Taking the square root of both sides gives: $v = \sin\theta \sqrt{\dfrac{gL}{\cos\theta}}$.

Related End-of-Chapter Exercises 21, 57.

Essential Question 5.6: If the speed of the ball in Example 5.6 is increased, what happens to *θ*, the angle between the string and the vertical?