

Answer to Essential Question 5.3: Yes. With the top object at rest on the inclined second object, give the top object a little push to get it moving. If it slides down the second object with constant velocity, the tangent of the angle of the incline equals the coefficient of kinetic friction.

5-4 A System of Two Objects and a Pulley

EXAMPLE 5.4 – Working with more than one object, and a pulley

A red box of mass $M = 10 \text{ kg}$ is placed on a ramp that is a 3-4-5 triangle, with a height of 3.0 m and a width of 4.0 m. The red box is tied to a green block of mass $m = 1.0 \text{ kg}$ by a string passing over a pulley, as shown in Figure 5.6. The coefficients of friction for the red box and the incline are $\mu_s = 0.50$ and $\mu_k = 0.25$. Use $g = 10 \text{ m/s}^2$. When the system is released from rest, what is the acceleration?

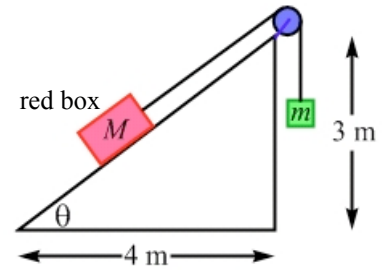


Figure 5.6: A diagram for the system of two objects and a pulley.

SOLUTION

To determine if there is an acceleration, and to find the direction of any acceleration, think about what happens if there is no friction. With no friction, we have the free-body diagrams in Figure 5.7. Choose a coordinate system aligned with the slope for the red box, with the positive x -direction down the slope and a positive y -direction perpendicular to the incline. If the red box moves down the slope, the green block moves up, so a consistent coordinate system for the green block has the positive direction up.

To align all the forces with the coordinate axes, only the force of gravity acting on the red box needs to be split into components. As shown in Figure 5.7c, we get $Mg \sin \theta$ directed down the slope and $Mg \cos \theta$ perpendicular to the slope.

Applying Newton's second law, $\sum \vec{F} = m\vec{a}$, to the green block gives:

$$+F_T - mg = +ma, \quad \text{which tells us that} \quad F_T = mg + ma.$$

Applying Newton's second law in the x -direction for the red box gives

$$+Mg \sin \theta - F_T = Ma, \quad \text{which gives} \quad Mg \sin \theta = F_T + Ma.$$

Combining the equation from the green block with the equation from the red box tells us that: $Mg \sin \theta = mg + ma + Ma$. This can be re-arranged into

$$(M \sin \theta - m)g = (m + M)a.$$

The acceleration of the system, with no friction, is positive if $M \sin \theta$ exceeds m . In this case, $M \sin \theta = 10 \text{ kg} \times (3/5) = 6.0 \text{ kg}$ is larger than the mass $m = 1.0 \text{ kg}$ of the green block. Thus, if the system accelerates, the red box accelerates down the slope. We don't know for sure that the system accelerates, because the force of static friction could prevent any motion. The force of static friction would be directed up the ramp, as in Figure 5.8, to stop the box from sliding down the ramp. How large must the force of static friction be to prevent the system from moving?

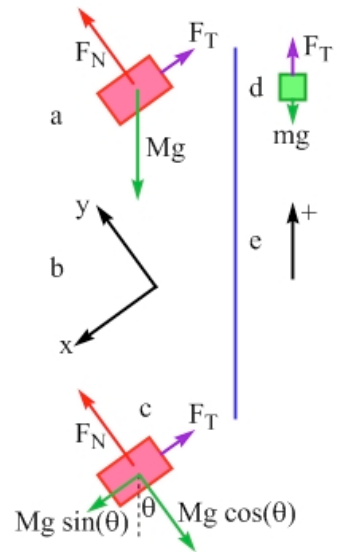


Figure 5.7: Free-body diagrams if there is no friction. (a) The free-body diagram of the red box. (b) An appropriate coordinate system for the red box. (c) The free-body diagram of the red box, with force components aligned with the coordinate system. (d) and (e), a free-body diagram and coordinate system for the green block.

If the system remains at rest, the acceleration is zero. Applying Newton's second law, $\sum \vec{F}_x = M\vec{a} = 0$, to the red box in the x -direction gives $+Mg \sin\theta - F_S - F_T = 0$.

With no acceleration, the force equation for the green block is $F_T = mg$. Using $F_T = mg$ in the previous equation gives $+Mg \sin\theta - F_S - mg = 0$. Thus, the force of static friction needed to prevent motion is:

$$F_S = Mg \sin\theta - mg = (M \sin\theta - m)g = [10 \text{ kg} \times (3/5) - 1.0 \text{ kg}]g = (5.0 \text{ kg})g.$$

Let's compare this value to the maximum possible force of static friction. Applying Newton's second law to the red box in the y -direction gives $F_N = Mg \cos\theta$, because there is no acceleration in that direction. Thus, the maximum possible force of static friction in this case is:

$$F_{S,\max} = \mu_s F_N = \mu_s Mg \cos\theta = 0.5 \times (10 \text{ kg}) \times (4/5) \times g = (4.0 \text{ kg})g.$$

The force of static friction required to prevent the system from accelerating is larger than its maximum possible value, which cannot happen. Thus, the system does accelerate. To find the acceleration, we again draw free-body diagrams, as in Figure 5.9. Now kinetic friction acts up the slope on the red box, and each object has a net force acting on it.

Applying Newton's second law, to the green block gives $+F_T - mg = +ma$, which tells us that $F_T = mg + ma$. A common error is to assume that $F_T = mg$, which is true only when the acceleration is zero.

Applying Newton's second law to the red box in the x -direction gives $+Mg \sin\theta - F_k - F_T = Ma$.

Substituting for the force of tension (using our result from analyzing the green block), we get $+Mg \sin\theta - F_k - mg - ma = Ma$, which we re-write as $+Mg \sin\theta - F_k - mg = (M + m)a$.

We can now substitute for the force of kinetic friction and find the acceleration:

$$a = \frac{+Mg \sin\theta - \mu_k F_N - mg}{M + m} = \frac{+Mg \sin\theta - \mu_k Mg \cos\theta - mg}{M + m}.$$

$$a = \frac{+10 \text{ kg} \times (10 \text{ m/s}^2)(3/5) - 0.25 \times (10 \text{ kg})(10 \text{ m/s}^2)(4/5) - (1.0 \text{ kg}) \times (10 \text{ m/s}^2)}{10.0 \text{ kg} + 1.0 \text{ kg}}.$$

$$a = \frac{+60 \text{ N} - 20 \text{ N} - 10 \text{ N}}{11.0 \text{ kg}} = \frac{+30 \text{ N}}{11.0 \text{ kg}} = +2.7 \text{ m/s}^2.$$

Note that the role of the pulley in this situation is simply to redirect the force of tension.

Related End-of-Chapter Exercises: 18, 41, 47.

Essential Question 5.4: If an object moves in a circle at constant speed, is there an acceleration?

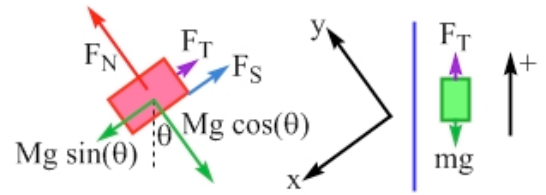


Figure 5.8: The free-body diagrams if static friction prevents motion are similar to those in Figure 5.7, except that, in this case, the forces balance and there is a force of static friction, on the red box, directed up the slope.

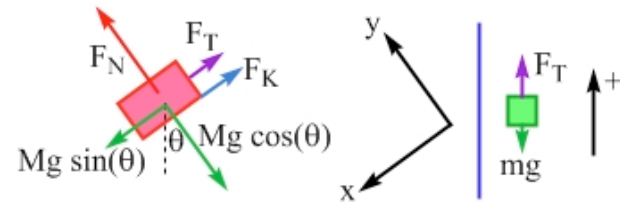


Figure 5.9: Free-body diagrams for the situation of the red box accelerating down the slope and the green block accelerating up.