

Answer to Essential Question 5.2: The two components of the contact force are the upward normal force and the horizontal force of kinetic friction. These two components are at right angles to one another, so we can use the Pythagorean theorem to find the magnitude of the contact force:

$$F_C = \sqrt{F_K^2 + F_N^2} = \sqrt{(16 \text{ N})^2 + (40 \text{ N})^2} = 43 \text{ N} .$$

5-3 Measuring the Coefficient of Friction

Let's connect the force ideas to the one-dimensional motion situations from Chapter 2.

EXPLORATION 5.3 – Measuring the coefficient of static friction

Coefficients of static friction for various pairs of materials are given in Table 5.1. Here's one method for experimentally determining these coefficients for a particular pair of materials. Take an aluminum block of mass m and a board made from a particular type of wood (we could also use a block of the wood and a piece of inflexible aluminum). Place the block on the board and slowly raise one end of the board. The angle of the board when the block starts to slide gives the coefficient of static friction. How?

To answer this question, let's extend the general method for solving a problem that involves Newton's laws.

Step 1 – Draw a diagram of the situation. The diagram is shown in Figure 5.5a.

Step 2 – Draw a free-body diagram of the block when it is at rest on the inclined board. Two forces act on the block, the downward force of gravity and the upward contact force from the board. We generally split the contact force into components, the normal force perpendicular to the incline, and the force of static friction acting up the slope. This free-body diagram is shown in Figure 5.5b

Step 3 – Choose an appropriate coordinate system. In this case, if we choose a coordinate system aligned with the board (one axis parallel to the board and the other perpendicular to it), as in Figure 5.5c, we will only have to split the force of gravity into components.

Step 4 – Split the force of gravity into components. If the angle between the board and the horizontal is θ , the angle between the force of gravity and the y -axis is also θ . The component of the force of gravity acting parallel to the slope has a magnitude of $F_{Gx} = F_G \sin\theta = mg \sin\theta$. The perpendicular component is $F_{Gy} = F_G \cos\theta = mg \cos\theta$.

These components are shown on the lower free-body diagram, in Figure 5.5d.

Step 5 – Apply Newton's second law twice, once for each direction. Again, we break a two-dimensional problem into two one-dimensional problems. With no acceleration in the y -direction we get: $\sum \vec{F}_y = m\vec{a}_y = 0$. Looking at the

lower diagram in Figure 5.5 for the y -direction forces:

$$+F_N - mg \cos\theta = 0 , \quad \text{which tells us that}$$

$$F_N = mg \cos\theta .$$

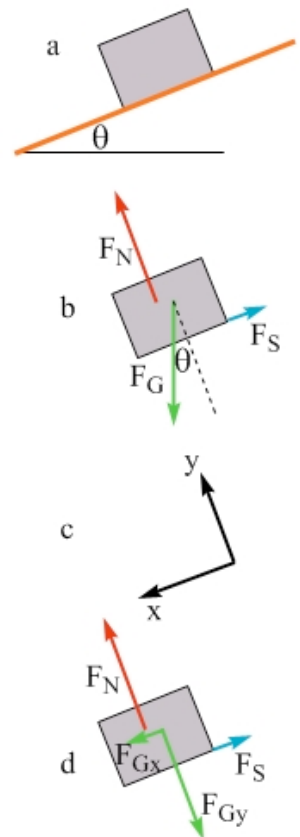


Figure 5.5: (a) A diagram showing the block on the board. (b) The initial free-body diagram of the block. (c) An appropriate coordinate system, aligned with the incline. (d) A second free-body diagram, with the forces aligned parallel to the coordinate axes.

While the box is at rest, there is no acceleration in the x -direction so: $\sum \vec{F}_x = m\vec{a}_x = 0$. Looking at the lower free-body diagram, Figure 5.5d, for the forces in the x -direction:

$$mg \sin\theta - F_S = 0, \quad \text{which tells us that } F_S = mg \sin\theta .$$

These two equations, $F_N = mg \cos\theta$ and $F_S = mg \sin\theta$, tell us a great deal about what happens as the angle of the incline increases. When the board is horizontal, the normal force is equal to mg and the static force of friction is zero. As the angle increases, $\cos\theta$ decreases from 1 and $\sin\theta$ increases from zero. Thus, as the angle increases, the normal force (and the maximum possible force of static friction) decreases, while the force of static friction required to keep the block at rest increases. At some critical angle θ_C , the force of static friction needed to keep the block at rest equals the maximum possible force of static friction. If the angle exceeds this critical angle, the block will slide. Using the definition of the coefficient of static friction:

$$\mu_s = \frac{F_{S,\max}}{F_N} = \frac{mg \sin\theta_C}{mg \cos\theta_C} = \tan\theta_C .$$

Thus, it is easy to determine coefficients of static friction experimentally. Take two objects and place one on top of the other. Gradually tilt the objects until the top one slides off. The tangent of the angle at which sliding occurs is the coefficient of static friction.

Key idea regarding the coefficient of static friction: The coefficient of static friction between two objects is the tangent of the angle beyond which one object slides down the other.

Related End-of-Chapter Exercises: 7, 36.

The steps we used to solve the problem in Exploration 5.3 can be applied generally to most problems involving Newton's laws. Let's summarize the steps here. Then, we will get some more practice applying the method in the next section.

A General Method for Solving a Problem Involving Newton's Laws in Two Dimensions

1. Draw a diagram of the situation.
2. Draw one or more free-body diagrams, with each free-body diagram showing all the forces acting on an object.
3. For each free-body diagram, choose an appropriate coordinate system. Coordinate systems for different free-body diagrams should be consistent with one another. A good rule of thumb is to align each coordinate system with the direction of the acceleration.
4. Break forces into components that are parallel to the coordinate axes.
5. Apply Newton's second law twice to each free-body diagram, once for each coordinate axis. Put the resulting force equations together and solve.

Related End-of-Chapter Exercises 17, 19.

Essential Question 5.3: Could we modify the procedure described in Exploration 5.3 to measure the coefficient of kinetic friction? If so, how could we modify it?