

5-1 Kinetic Friction

When two objects are in contact, the friction force, if there is one, is the component of the contact force between the objects that is parallel to the surfaces in contact. (The component of the contact force that is perpendicular to the surfaces is the normal force.) Friction tends to oppose relative motion between objects. When there is relative motion, the friction force is the kinetic force of friction. For instance, when a book slides across a table, kinetic friction slows the book.

If there is relative motion between objects in contact, the force of friction is the kinetic force of friction (F_K). We will use a simple model of friction that assumes the force of kinetic friction is proportional to the normal force. A dimensionless parameter, called the coefficient of kinetic friction, μ_K , represents the strength of that frictional interaction.

$$\mu_K = \frac{F_K}{F_N} \quad \text{so} \quad F_K = \mu_K F_N \quad \text{(Equation 5.1: Kinetic friction)}$$

Note that some typical values for the coefficient of kinetic friction, as well as for the coefficient of static friction, which we will define in Section 5-2, are given in Table 5.1. The coefficients of friction depend on the materials that the two surfaces are made of, as well as on the details of their interaction. For instance, adding a lubricant between the surfaces tends to reduce the coefficient of friction. There is also some dependence of the coefficients of friction on the temperature.

Interacting materials	Coefficient of kinetic friction (μ_K)	Coefficient of static friction (μ_S)
Rubber on dry pavement	0.7	0.9
Steel on steel (unlubricated)	0.6	0.7
Rubber on wet pavement	0.5	0.7
Wood on wood	0.3	0.4
Waxed ski on snow	0.05	0.1
Friction in human joints	0.01	0.01

Table 5.1: Approximate coefficients of kinetic friction, and static friction (see Section 5-2), for various interacting materials.

EXPLORATION 5.1 – A sliding book

You slide a book, with an initial speed v_i , across a flat table. The book travels a distance L before coming to rest. What determines the value of L in this situation?

Step 1 – Sketch a diagram of the situation and a free-body diagram of the book. These diagrams are shown in Figure 5.1.

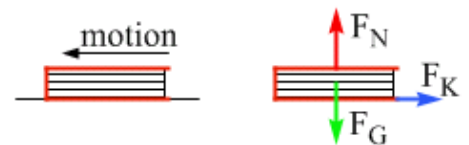


Figure 5.1: A diagram of the sliding book and a free-body diagram showing the forces acting on the book as it slides.

The Earth applies a downward force of gravity on the book, while the table applies a contact force. Generally, we split the contact force into components and show an upward normal force and a horizontal force of kinetic friction, \vec{F}_K , acting on the book. \vec{F}_K points to the right on the book, opposing the relative motion between the book (moving left) and the table (at rest).

Step 2 – Find an expression for the book’s acceleration. We are working in two dimensions, and a coordinate system with axes horizontal and vertical is convenient. Let’s choose up to be positive for the y -axis and, because the motion of the book is to the left, let’s choose left to be the positive x -direction. We will apply Newton’s Second Law twice, once for each direction. There is no acceleration in the y -direction, so we have: $\sum \vec{F}_y = m\vec{a}_y = 0$.

Because we’re dealing with the y sub-problem, we need only the vertical forces on the free-body diagram. Adding the vertical forces as vectors (with appropriate signs) tells us that:

$$+F_N - F_G = 0, \quad \text{so} \quad F_N = F_G = mg.$$

Repeat the process for the x sub-problem, applying Newton’s Second Law, $\sum \vec{F}_x = m\vec{a}_x$. Because the acceleration is entirely in the x -direction, we can replace \vec{a}_x by \vec{a} . Now, we focus only on the forces in the x -direction. All we have is the force of kinetic friction, which is to the right while the positive direction is to the left. Thus: $-F_K = m\vec{a}$.

We can now solve for the acceleration of the book, which is entirely in the x -direction:

$$\vec{a} = \frac{-F_K}{m} = \frac{-\mu_K F_N}{m} = \frac{-\mu_K mg}{m} = -\mu_K g.$$

Step 3 – Determine an expression for length, L , in terms of the other parameters. Let’s use our expression for the book’s acceleration and apply the method for solving a constant-acceleration problem. Table 5.2 summarizes what we know. Define the origin to be the book’s starting point and the positive direction to be the direction of motion.

We can use equation 2.10 to relate the distance traveled to the coefficient of friction:

$$v^2 = v_i^2 + 2a\Delta x, \quad \text{so} \quad \Delta x = \frac{v^2 - v_i^2}{2a}.$$

In this case, we get
$$L = \frac{0 - v_i^2}{-2\mu_K g} = \frac{v_i^2}{2\mu_K g}.$$

Initial position	$x_i = 0$
Final position	$x = +L$
Initial velocity	$+v_i$
Final velocity	$v = 0$
Acceleration	$a = -\mu_K g$

Table 5.2: A summary of what we know about the book’s motion to help solve the constant-acceleration problem.

Let’s think about whether the equation we just derived for L makes sense. The equation tells us that the book travels farther with a larger initial speed or with smaller values of the coefficient of friction or the acceleration due to gravity, which makes sense. It is interesting to see that there is no dependence on mass – all other parameters being equal, a heavy object travels the same distance as a light object.

Key idea: A useful method for solving a problem with forces in two dimensions is to split the problem into two one-dimensional problems. Then, we solve the two one-dimensional problems individually. **Related End-of-Chapter Exercises 13 – 15, 31.**

Essential Question 5.1: You are standing still and you then start to walk forward. Is there a friction force involved here? If so, is it the kinetic force of friction or the static force of friction?