Answer to Essential Question 4.7 Because the ball has constant velocity in the *x* direction, the graphs for the *x* components of the ball's motion match the constant-velocity graphs from Chapter 2. Similarly, because the ball has constant acceleration in the *y* direction, the graphs for the *y* components of the ball's motion match the constant-acceleration graphs from Chapter 2.

4-8 Range, and One Final Example

The **range** of a projectile is the horizontal distance it travels before striking the ground. If a projectile lands at the same height from which it was launched, what launch angle gives the maximum range if the initial speed of the projectile is constant? Let's answer this question with the aid of the answer to Essential Question 4.6, that the time-of-flight when a projectile lands at the height from which it was launched is $t_{\text{impact}} = 2v_{iy} / g$. Substituting this time into equation 4.3x, taking the origin to be the launch point and recalling that the acceleration is zero, gives:

$$\vec{x} = \vec{x}_i + \vec{v}_{ix}t + \frac{1}{2}\vec{a}_x t^2 = 0 + \vec{v}_{ix}t + 0 = \vec{v}_{ix}t \;.$$

Therefore, $\text{range} = v_{ix}\, t_{\text{impact}} = \dfrac{2v_{ix}v_{iy}}{g}$.

If the launch angle θ is measured from the horizontal, we have $v_{ix} = v_i \cos\theta$ and $v_{iy} = v_i \sin\theta$. Substituting these expressions into the range equation above gives:

$$\text{range} = \frac{2v_{ix}v_{iy}}{g} = \frac{2(v_i\cos\theta)(v_i\sin\theta)}{g} = \frac{2v_i^2 \sin\theta\,\cos\theta}{g} \,.$$

This can be simplified with the trigonometric identity $2\sin\theta\,\cos\theta = \sin(2\theta)$:

$$\text{range} = \frac{v_i^2 \sin(2\theta)}{g} \,.$$

This equation applies only when the projectile lands at the height from which it was launched.

So, the range is proportional to the square of the initial speed, is inversely proportional to *g*; and depends on the launch angle in an interesting way. A graph of $\sin(2\theta)$ is shown in Figure 4.18. If the launch angle θ is between 0 and 90˚, then 2θ is between 0 and 180˚. Keeping v_i and *g* constant, the maximum range is achieved when $\sin(2\theta)$ is maximized. This occurs when $2\theta = 90˚$, so $\theta = 45˚$ is the launch angle that gives the maximum range when the projectile lands at the same height from which it was launched.

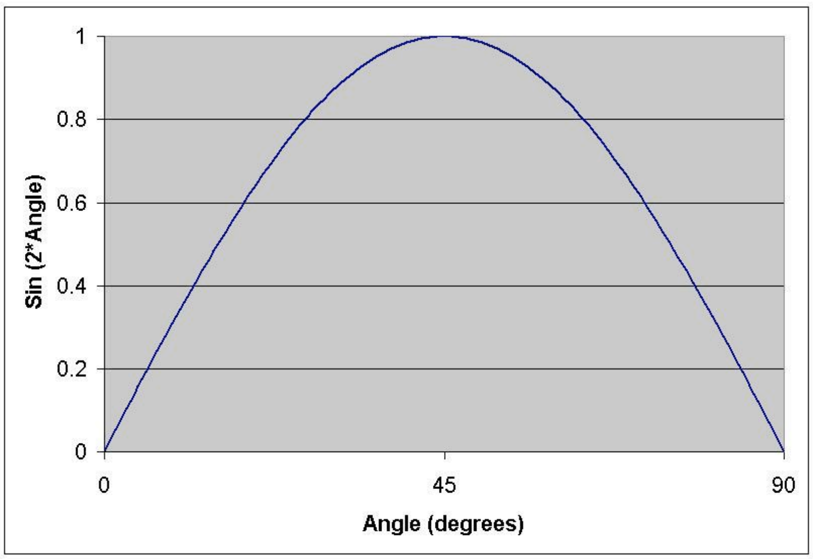

Figure 4.18: A graph of $\sin(2\theta)$ as a function of θ.

Related End-of-Chapter Exercises: 57 – 62.

EXAMPLE 4.8 – A corner kick

A corner kick, in which a player directs the ball into a crowd of players in front of the net, is one of the most exciting plays in soccer. You are taking the kick, and you want the ball, while it is on the way down, to be precisely 2.00 m above the ground when it reaches your teammate, who will attempt to head the ball into the net. Your teammate is 30.0 m away, and you kick the ball from ground level at an angle of 25.0° with respect to the horizontal. What initial speed should you give the ball? Use $g = 9.81$ m/s^2. (a) Sketch a diagram of the situation, and organize what you know in a data table, keeping the x (horizontal) information separate from the y (vertical) information. (b) Use the x information to find an expression for the time it takes the ball to reach your teammate. (c) Use the y information, and the time, to find the ball's initial speed.

SOLUTION (a) Define the origin as the launch point, and the positive directions so that toward your teammate is the positive x direction, and up is the positive y direction. Figure 4.19 shows a diagram of the situation, and the data is given in Table 4.7.

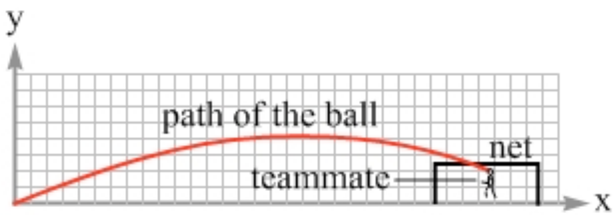

Figure 4.19: The flight of the soccer ball. The squares on the grid measure 1 m × 1 m.

Component	x-direction	y-direction
Initial position	$x_i = 0$	$y_i = 0$
Final position	$x_f = +30.0$ m	$y_f = +2.00$ m
Initial velocity	$v_{ix} = +v_i \cos(25°)$	$v_{iy} = +v_i \sin(25°)$
Acceleration	$a_x = 0$	$a_y = -9.81$ m/s^2

Table 4.7: Organizing the data for the problem.

(b) Here, we can use Equation 4.3x, $x = x_i + v_{ix}t + (0.5)a_x t^2$, which reduces to $x = v_{ix}t$. Solving for the time the ball takes to reach your teammate, we get: $t = \dfrac{x}{v_{ix}} = \dfrac{+30.0 \text{ m}}{v_i \cos(25°)}$.

(c) We can now substitute our expression for time into equation 4.3y:

$$y = y_i + v_{iy}t + \frac{1}{2}a_y t^2 : \qquad +2.00 \text{ m} = v_i \sin(25°) \times \left(\frac{+30.0 \text{ m}}{v_i \cos(25°)}\right) + \frac{1}{2}(-9.81 \text{ m/s}^2) \times \left(\frac{+30.0 \text{ m}}{v_i \cos(25°)}\right)^2 .$$

Simplifying, we get: $+2.00 \text{ m} = +13.989 \text{ m} - \dfrac{5374.40 \text{ m}^3/\text{s}^2}{\text{v}_\text{i}^2}$.

This gives $+\dfrac{5374.40 \text{ m}^3/\text{s}^2}{\text{v}_\text{i}^2} = +11.989 \text{ m}$, so $v_i = \sqrt{\dfrac{5374.40 \text{ m}^3/\text{s}^2}{11.989 \text{ m}}} = 21.2 \text{ m/s}$.

This is fast, but not unusual for a typical soccer game.

Related End-of-Chapter Exercises: 19, 28, 63.

Essential Question 4.8 Return again to the situation described in Example 4.8. Could we have found the answer more easily by applying the range equation from the previous page?