Answer to Essential Question 4.5 Yes. From your perspective, moving with the sailboat at a constant velocity, the beanbag still drops straight down from rest. From the perspective of someone at rest on shore, the beanbag follows a parabolic motion. Its horizontal motion keeps it over the bucket at all times, though, so it still falls into the bucket because the bucket, beanbag, and boat all have the same horizontal velocity. We're neglecting air resistance here, by the way.

4-6 An Example of Projectile Motion

When a soccer goalkeeper comes off the goal line to challenge a shooter, the shooter can score by kicking the ball so that it goes directly over the goalkeeper, and then comes down in time to either bounce into, or fly into, the net. This is known as chipping the goaltender, because the shooter chips the ball to produce the desired effect. Let's analyze this situation in detail.

EXAMPLE 4.6 – Chipping the goaltender
Precisely 1.00 s after you kick it from ground level, a soccer ball passes just above the outstretched hands of a goaltender who is 5.00 m away from you, and lands on the goal line before bouncing into the net for the winning goal in a soccer match. The goaltender's fingertips are 2.50 m above the ground. Neglect air resistance, and use $g = 9.81$ m/s^2.

(a) At what angle did you launch the ball?
(b) What was the ball's initial speed?
(c) Assuming the ground is completely level, how long does the ball spend in the air?

SOLUTION
As usual, we should be methodical. A diagram of the situation is shown in Figure 4.14. The diagram includes an appropriate coordinate system, with the origin at the launch point. Table 4.6 summarizes what we know, separated into x and y components.

Take the direction of the horizontal component of the ball's motion to be the positive x direction, and take up to be the positive y direction.

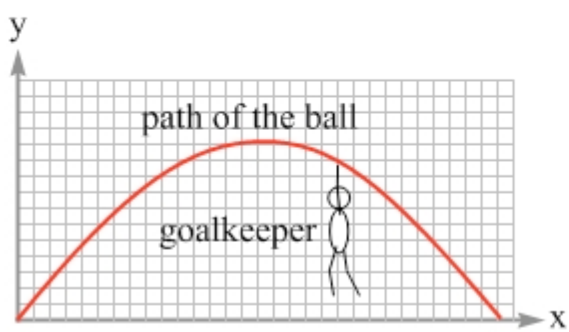

Figure 4.14: The flight of the soccer ball. The squares on the grid measure 0.25 m × 0.25 m.

Component	*x* direction	*y* direction
Initial position	$x_i = 0$	$y_i = 0$
Final position	$x = ?$	$y = 0$
Initial velocity	$v_{ix} = ?$	$v_{iy} = ?$
Final velocity	$v_x = v_{ix}$	$v_y = ?$
Acceleration	$a_x = 0$	$a_y = -g$
At t = 1.00 s	$x_{t=1} = +5.00$ m	$y_{t=1} = +2.50$ m

Table 4.6: Organizing the data for the problem.

(a) ***At what angle did you launch the ball?*** It is tempting to draw a right-angled triangle with a base of 5.00 m and a height of 2.50 m and take the angle between the base and the hypotenuse to be the launch angle, but doing so is incorrect. The ball follows a parabolic path that curves down, not a straight path to just above the goaltender's fingertips. Thus, the launch angle is larger than the angle of that particular right-angled triangle.

The launch angle is the angle between the ball's initial velocity and the horizontal, so let's work on determining the initial velocity. We can use what we know about the ball at t = 1.00 s to help us. Once again, we will make use of the equations in Table 4.4.

To find the x component of the initial velocity, use equation 4.3x: $x_{t=1} = x_i + v_{ix}t + \frac{1}{2}a_x t^2$.

$$x_{t=1} = 0 + v_{ix}\,t + 0 .$$

This gives $v_{ix} = \frac{x_{t=1}}{t} = \frac{+5.00 \text{ m}}{1.00 \text{ s}} = +5.00 \text{ m/s}$.

To find the y component of the initial velocity, use equation 4.3y: $y_{t=1} = y_i + v_{iy}t + \frac{1}{2}a_y t^2$.

$$y_{t=1} = 0 + v_{iy}t - \frac{1}{2}gt^2 .$$

This gives $\vec{v}_{iy} = \frac{y_{t=1} + gt^2/2}{t} = \frac{+2.50 \text{ m} + 4.90 \text{ m}}{1.00 \text{ s}} = +7.40 \text{ m/s}$.

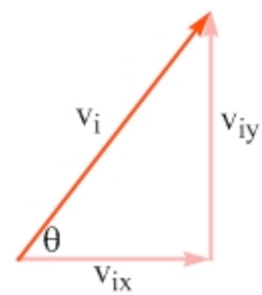

Figure 4.15: The right-angled triangle that we use to find the launch angle and launch speed of the ball.

From the two components of the initial velocity, we can determine the launch angle θ and the initial speed. The geometry of the situation is shown in Figure 4.15.

$$\tan\theta = \frac{v_{iy}}{v_{ix}} = \frac{7.40 \text{ m/s}}{5.00 \text{ m/s}}, \text{ so } \theta = 56.0^\circ .$$

(b) ***What was the initial speed of the ball?*** The launch speed is the magnitude of the initial velocity. Applying the Pythagorean theorem to the triangle in Figure 4.15 gives:

$$v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{(5.00 \text{ m/s})^2 + (7.40 \text{ m/s})^2} = 8.93 \text{ m/s} .$$

(c) ***Assuming the ground is completely level, how long does the ball spend in the air?*** We should not assume that the ball passes over the goaltender when the ball is at its maximum height. Here is a good rule of thumb: If you do not have to assume something, don't assume it! Instead, let's make use of Equation 4.3y to determine the time for the entire flight:

$$y = y_i + v_{iy}t + \frac{1}{2}a_y t^2 .$$

$$0 = 0 + v_{iy}t - \frac{1}{2}gt^2.$$

At first glance it looks as though we have to use the quadratic formula to solve this equation, but we can simply divide through by a factor of t. Doing so gives:

$$0 = +v_{iy} - \frac{1}{2}gt .$$

Solving for t gives $t = \frac{2v_{iy}}{g} = \frac{2\times 7.405 \text{ m/s}}{9.81 \text{ m/s}^2} = \frac{14.81 \text{ m/s}}{9.81 \text{ m/s}^2} = 1.51 \text{ s}$.

Related End-of-Chapter Exercises: 13, 17, 18.

Essential Question 4.6 Consider again the ball from Example 4.6. How does the time it takes the ball to reach maximum height compare to the time for the entire flight? Explain.