***Answer to Essential Question 4.3*** The plane's velocity relative to the ground is the vector sum of the plane's velocity relative to the air (150 km/h north) and the air's velocity relative to the ground (50 km/h west). This sum is less than 200 km/h in a particular direction because the two vectors are not in the same direction. The plane's speed relative to the ground is 158 km/h, the length of the hypotenuse of the right-angled triangle we get by adding the vectors.

## 4-4 Projectile Motion

Projectile motion is, in general, two-dimensional motion that results from an object with an initial velocity in one direction experiencing a constant force in a different direction. A good example is a ball you throw to a friend. You give the ball an initial velocity when you throw it, and then the force of gravity acts on the ball as it travels to your friend. In this section, we will learn how to analyze this kind of situation.

**EXPLORATION 4.4 – A race**
You release one ball (ball *A*) from rest at the same time you throw another ball (ball *B*), which you release with an initial velocity that is directed entirely horizontally. You release both balls simultaneously from the same height *h* above level ground. Neglect air resistance.

**Step 1** - ***Which ball travels a greater distance before hitting the ground?*** Ball *A* takes the shortest path to the ground, so ball *B* travels farther.

**Step 2** - ***Which ball reaches the ground first? Why?*** We can construct a motion diagram (see Figure 4.9) by, for instance, analyzing a video of the balls as they fall. Many people think that because ball *B* travels farther it takes longer to reach the ground; however, ball *B* also has a higher speed. The reality is that both balls reach the ground at the same time. The reason is that the motion of ball *B* can be viewed as a combination of its horizontal motion and its vertical motion. The horizontal motion has no effect whatsoever on the vertical motion, so what happens vertically for ball *B* is exactly the same as what happens vertically for ball *A*.

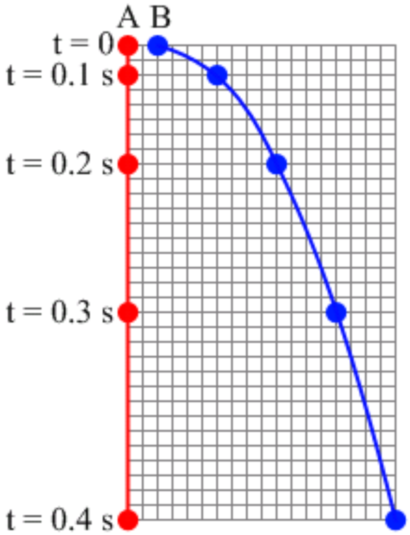

**Figure 4.9**: A motion diagram can be constructed from experimental evidence, such as by analyzing a video of the balls as they fall.

**Key idea for projectile motion**: The key idea of this chapter is the independence of *x* and *y*. The basic idea is that the motion that happens in one direction (*x*) is independent of the motion that happens in a perpendicular direction (*y*), and vice versa, as long as the force is constant.
**Related End-of-Chapter Exercises: 9, 10.**

The *x*-direction and *y*-direction motions are independent in the sense that each of the one-dimensional motions occurs as if the other motion is not happening. These motions are connected, though. The object's motion generally stops after a particular time, so the time is the same for the *x*-direction motion and the *y*-direction motion.

This powerful concept allows us to treat a two-dimensional projectile motion problem as two separate one-dimensional problems. We already have a good deal of experience with one-dimensional motion, so we can build on what we learned in Chapter 2. For the most part, we will deal with situations where the acceleration is constant, so all our experience with constant-acceleration situations in one dimension will be directly relevant here.

### Solving a Two-Dimensional Constant-Acceleration Problem

Our general method for analyzing a typical projectile-motion problem builds on the method we used for analyzing one-dimensional constant-acceleration motion in Chapter 2. The basic idea is to split the two-dimensional problem into two one-dimensional subproblems, which we can call the *x* subproblem and the *y* subproblem.

1. Draw a diagram of the situation.
2. Draw a free-body diagram of the object showing all the forces acting on the object while it is in motion. A free-body diagram helps in determining the acceleration of the object.
3. Choose an origin.
4. Choose an *x*-*y* coordinate system, showing which way is positive for each coordinate axis.
5. Organize your data, keeping the information for the *x* subproblem separate from the information for the *y* subproblem.
6. Only then should you turn to the constant-acceleration equations. Make sure the acceleration is constant so the equations apply! We use the same three equations that we used in Chapter 2, but we customize them for the *x* and *y* subproblems, as follows:

| Equation from Chapter 2 | *x*-direction equations | *y*-direction equations |
|---|---|---|
| $v = v_i + at$ (2.7) | $v_x = v_{ix} + a_x t$ (4.2x) | $v_y = v_{iy} + a_y t$ (4.2y) |
| $x = x_i + v_i t + \frac{1}{2}at^2$ (2.9) | $x = x_i + v_{ix}t + \frac{1}{2}a_x t^2$ (4.3x) | $y = y_i + v_{iy}t + \frac{1}{2}a_y t^2$ (4.3y) |
| $v^2 = v_i^2 + 2a\Delta x$ (2.10) | $v_x^2 = v_{ix}^2 + 2a_x \Delta x$ (4.4x) | $v_y^2 = v_{iy}^2 + 2a_y \Delta y$ (4.4y) |

**Table 4.4**: Constant acceleration equations for two-dimensional projectile motion. The equation numbers are shown in parentheses after each equation.

Let's apply the method above to analyze the race from Exploration 4.4. We begin by sketching a motion diagram and a free-body diagram for each ball, and continue the analysis in the next section. On the motion diagram for ball *B*, show the separate *x* (horizontal) and *y* (vertical) motions.

The motion diagrams are shown in Figure 4.10, while the free-body diagram of each ball is shown in Figure 4.11. Let's consider the motion from just after the balls are released until just before the balls make contact with the ground. Because the only force acting on either ball is the force of gravity, the same free-body diagram applies to both objects.

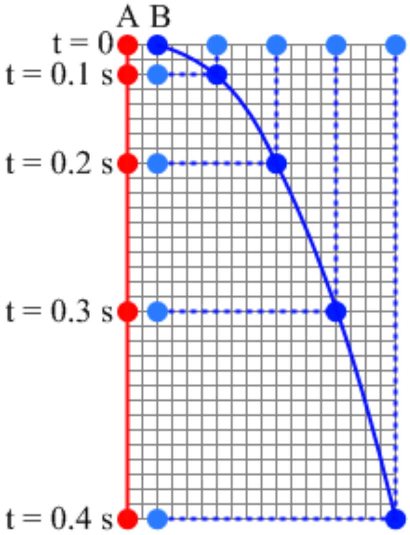

**Figure 4.10**: Motion diagram for balls *A* and *B*. For ball *B*, the vertical and horizontal motions are shown separately. These two independent motions combine to give the parabolic path followed by ball *B*.

***Essential Question 4.4*** The free-body diagrams in Figure 4.11 imply that the balls have the same mass. What would happen if the balls had different masses?

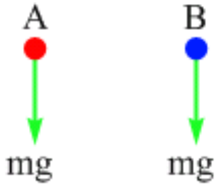

**Figure 4.11**: Free-body diagrams for balls *A* and *B*. From the instant just after you release the balls until the instant just before the balls hit the ground, the only force acting on either ball is the force of gravity, so the balls have identical free-body diagrams. The balls travel along different paths only because their initial velocities are different.