Answer to Essential Question 4.2 Running on the moving sidewalk is a disadvantage for Mia because she spends considerably more time running against the moving sidewalk than she does running with it.

4-3 Relative Velocity in Two Dimensions

Let's modify Exploration 4.1, about your motion on a ferry, to see how things change when we switch to two dimensions. The major difference between one and two dimensions is that it is more challenging to add vectors in two dimensions than in one dimension; however, we simply need to follow what we learned about vector addition in Chapter 1.

EXPLORATION 4.3 – Crossing a river
You are crossing a river on a ferry. The ferry is pointing north, and it is traveling at a constant speed of 7.0 m/s relative to the water. The current in the river is 2.0 m/s directed southeast.

Step 1 - ***What is the ferry's velocity relative to the shore?*** The first step is to draw a vector diagram, as in Figure 4.7, showing how the relevant vectors combine to produce the velocity we're interested in.

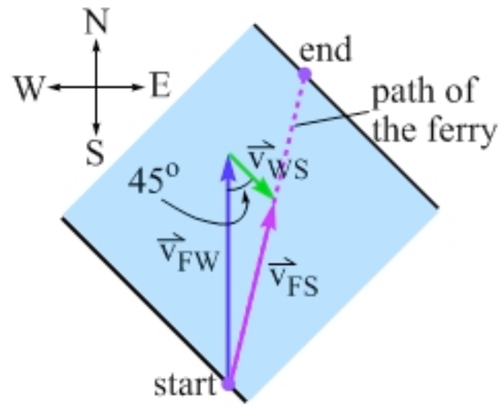

Figure 4.7: Vector diagram to find the ferry's velocity relative to the shore. We add the velocity of the ferry relative to the water to the velocity of the water relative to the shore to find the velocity of the ferry relative to the shore.

We can apply the standard relative-velocity equation, using subscripts of F for the ferry, W for the water, and S for the shore, to get:

$$\vec{v}_{FS} = \vec{v}_{FW} + \vec{v}_{WS} .$$

This equation is consistent with the vector diagram in Figure 4.7. Because the direction of the current is specified as southeast, we can use a 45° angle for that vector.

Here we have a typical vector-addition problem in two dimensions. One method of solving the problem is to break the vectors into components and add the components. This method involves separating the two-dimensional problem into two different one-dimensional problems, which we do quite often in physics.

As we learned in Chapter 1, we can put together a table to help us keep the *x* and *y* components separate. Doing so makes it easy to add the vectors using the component method. Define the positive *x* direction as east, and the positive *y* direction as north.

Vector	***x* components**	***y* components**
$\vec{v}_{FW}$	$v_{FWx} = 0$	$v_{FWy} = +7.00$ m/s
$\vec{v}_{WS}$	$v_{WSx} = +(2.00 \text{ m/s})\cos(45^\circ) = +1.41$ m/s	$v_{WSy} = -(2.00 \text{ m/s})\sin(45^\circ) = -1.41$ m/s
$\vec{v}_{FS} = \vec{v}_{FW} + \vec{v}_{WS}$	$v_{FSx} = v_{FWx} + v_{WSx}$ $v_{FSx} = 0 + 1.41 \text{ m/s} = +1.41$ m/s	$v_{FSy} = v_{FWy} + v_{WSy}$ $v_{FSy} = +7.00 \text{ m/s} - 1.41 \text{ m/s} = +5.59$ m/s

Table 4.3: Adding the vectors using components.

We can leave the answer in terms of components, as is done at the bottom of Table 4.3, or we can specify the velocity of the ferry with respect to the shore in terms of its magnitude and direction. To specify the velocity vector in the magnitude-direction format, start by drawing the right-angled triangle corresponding to the vector and its components, as in Figure 4.8.

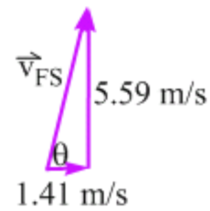

Figure 4.8: A vector diagram showing how the components add to give the net velocity vector.

The magnitude of the vector can be found using the Pythagorean theorem:

$$v_{FS} = \sqrt{1.41^2 + 5.59^2} = 5.8 \text{ m/s}.$$

The angle can be found using $\tan\theta = \frac{5.59}{1.41}$, so $\theta = 76^\circ$.

This angle is measured with respect to east. The ferry is not traveling due east, but rather somewhat north of east. Thus, we can say that the ferry's velocity relative to the shore is 5.8 m/s at an angle of 76˚ north of east (or, equivalently, 14˚ east of north).

Step 2 - ***You are standing on an observation deck at the top of the ferry, leaning against the railing on the starboard (right) side of the ferry. Suddenly someone yells from the port (left) side that there are some porpoises frolicking in the water. To get from one side of the ferry to the other in the shortest time to see the porpoises, in which direction should you run? Choose the best answer below.***

1. Run directly across the ferry, taking the shortest route to the other side (this means you will be carried along with the ferry, and the current).
2. Run partly toward the bow (front) of the ferry, so the ferry's velocity adds to your velocity and you travel at a higher speed.
3. Run partly toward the stern (rear) of the ferry, so the distance you travel relative to the shore is minimized.

If the ferry is docked and you want to cross from one side of the ferry to the other in the least time, you simply head straight across the ferry. If the ferry is moving, you do the same thing! The fact that the ferry is moving does not even cross your mind, which is good, because that is irrelevant. The reason the ferry's motion does not matter is that the ferry's motion is perpendicular to the direction you want to go, and a velocity in one direction does not affect motion in a perpendicular direction. The time for you to cross from one side of the ferry to the other is determined by how much of your velocity is directed across the ferry. If you aim yourself entirely in that direction, you minimize the time it takes to cover a particular distance in that direction.

Key ideas for relative velocity in two dimensions: The relative velocity equation, $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$, applies just as well in two or three dimensions as it does in one dimension. To solve the equation, we can apply the vector-addition methods, such as the component method, we covered in Chapter 1. Another important concept is that motion in two dimensions can be split into two independent one-dimensional motions. **Related End-of-Chapter Exercises: 7, 8.**

Essential Question 4.3 A pilot has aimed her plane north, and her airspeed indicator reads 150 km/h. The control tower reports that the wind is directed west at 50 km/h. Explain why the plane's speed relative to the ground is greater than 150 km/h, but less than 200 km/h.