Answer to Essential Question 4.1 Yes. Adding an additional velocity of 2.0 m/s directed south will cancel the velocity of 2.0 m/s north that you are moving with respect to the shore. You were already walking at 3.0 m/s south with respect to the ferry, so you now need to be moving at 5.0 m/s south (so you need to run now) with respect to the ferry.

4-2 Combining Relative Velocity and Motion

Let's now connect relative velocity with the one-dimensional motion situations that we considered in Chapter 2.

EXPLORATION 4.2 – Who's faster?
On their way to play soccer in the World Cup, Mia and Brandi get stranded at O'Hare Airport in Chicago because of bad weather. Late at night, with nobody else around, they decide to have a race to see who is faster. They run down to a particular point, then turn around and return to their starting point. That race turns out to be a tie.

They race again, this time with Mia running on a moving sidewalk. They start at the same time, run east at top speed a distance L through the airport terminal, and then turn around and run west back to the starting point. Brandi runs at a constant speed v, while Mia runs on a moving sidewalk that travels at a speed of $v/2$. Mia runs at a constant speed v relative to the moving sidewalk. Neglect the time it takes the women to turn around at the halfway point. A diagram is shown in Figure 4.4 to help with the analysis.

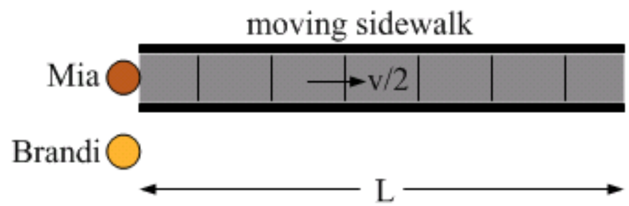

Figure 4.4: A diagram showing the positions of Mia and Brandi at the start of the race.

Step 1 - ***Make a prediction – who wins this race?*** Most people predict that this race is also a tie, because Mia is helped by the moving sidewalk for half the distance, and she has to work against the moving sidewalk for the remainder of the race.

Step 2 - ***If Brandi takes a time T to reach the turn-around point, how long does Mia take to reach the same point?*** First, let's do a relative-velocity analysis for Mia as she runs east, treating east as the positive direction. Figure 4.5 shows the vectors being added together. Using subscripts of M for Mia, S for sidewalk, and G for ground, we get:

$$\vec{v}_{MG} = \vec{v}_{MS} + \vec{v}_{SG} = +v + \frac{v}{2} = +\frac{3v}{2}$$

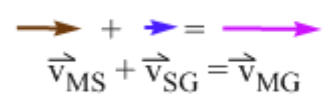

Figure 4.5: A vector diagram showing Mia's velocity relative to the ground as she runs east.

This relative-velocity situation is really a one-dimensional motion problem in disguise. Let's analyze the motion as the two women are moving away from the start line, choosing the origin as the start line and the positive direction to the right, in the direction the women are running. Let's summarize what we know in Table 4.1.

	Mia	**Brandi**
Initial position	$x_{iM} = 0$	$x_{iB} = 0$
Final position	$x_{fM} = +L$	$x_{fB} = +L$
Initial velocity	$v_{iM} = +3v/2$	$v_{iB} = +v$
Acceleration	$a_M = 0$	$a_B = 0$
Time	$t_M = ?$	$t_B = T$

Table 4.1: Organizing the data for the outbound trips.

Let's analyze Brandi's motion to see how the time T relates to the distance L and the speed v. We can use the following constant-acceleration equation, which we used previously in Section 2-5, to relate these parameters:

$$x_{fB} = x_{iB} + v_{iB}t_B + \frac{1}{2}a_B t_B^2 .$$

Substituting appropriate values from Table 4.1 gives: $L = 0 + vT + 0$, and thus $T = L / v$.

Using a similar analysis for Mia, we start with: $x_{fM} = x_{iM} + v_{iM}t_M + \frac{1}{2}a_M t_M^2$.

Substituting appropriate values from Table 4.1, we get:

$$L = 0 + \frac{3}{2}vt_M + 0 \text{, which gives } t_M = \frac{2L}{3v} = \frac{2}{3}T.$$

So Mia has a sizable lead by the time she reaches the turn-around point.

Step 3 - *Brandi takes an equal time T for the return trip from the turn-around point to the start/finish line. How long does Mia's return trip take?* The vector addition in this case is shown in Figure 4.6. For Mia's return trip, her velocity relative to the ground is:

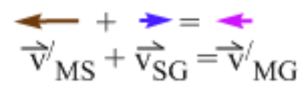

Figure 4.6: A vector diagram showing Mia's velocity relative to the ground as she runs west.

$$\vec{v}'_{MG} = \vec{v}'_{MS} + \vec{v}_{SG} = -v + \frac{v}{2} = -\frac{v}{2} .$$

The primed ($'$) values represent the values of these variables on the return trip.

The data table for the return trips is shown in Table 4.2. For Brandi, the return trip is the same as the outbound trip, so there is no need to repeat that analysis. Let's solve for Mia's time for the return trip:

$$x'_{fM} = x'_{iM} + v'_{iM}t_M + \frac{1}{2}a'_M (t'_M)^2 .$$

	Mia	Brandi
Initial position	$x'_{iM} = +L$	$x'_{iB} = +L$
Final position	$x'_{fM} = 0$	$x'_{fB} = 0$
Initial velocity	$v'_{iM} = -v/2$	$v'_{iB} = -v$
Acceleration	$a'_M = 0$	$a'_B = 0$
Time	$t'_M = ?$	$t'_B = T$

Table 4.2: Organizing the data for the return trips.

Substituting the values from Table 4.2 gives:

$$0 = +L - \frac{1}{2}vt'_M + 0 \text{, which gives } t'_M = \frac{2L}{v} = 2T.$$

Step 4 - *Based on your answers to steps 2 and 3, who wins the race?* Brandi's time for the entire trip is $2T$, the same time Mia takes just to come back along the moving sidewalk. Mia's total time is $2T + 2T/3$, so Brandi wins this race quite easily.

Key ideas: The methods we used to analyze one-dimensional motion situations in Chapter 2 can be combined with relative-velocity problems. **Related End-of-Chapter Exercises: 2, 31, 32.**

Essential Question 4.2 In Exploration 4.2, we concluded that Mia is at a disadvantage in the race when she runs on the moving sidewalk. What is a good conceptual explanation for this disadvantage?