4-1 Relative Velocity in One Dimension

Before we generalize to two dimensions, let's consider a familiar situation involving relative velocity in one dimension. You are driving east along the highway at 100 km/h. The car in the next lane looks like it is barely moving relative to you, while a car traveling in the opposite direction looks like it is traveling at 200 km/h. This is your perception, even though the speedometers in all three vehicles say that each car is traveling at about 100 km/h.

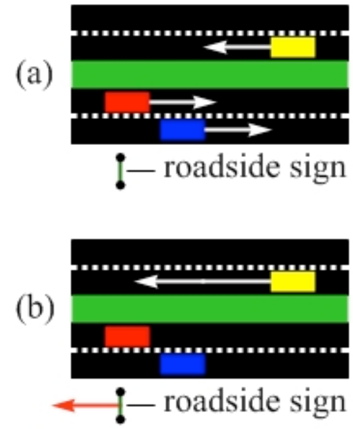

Figure 4.1: (a) An overhead view, with arrows showing the velocities of three cars and a roadside sign, with respect to the ground. (b) The same situation, but with the velocities shown with respect to you, the driver of the bottom car.

How can we explain your observations? First, consider the velocity of your car relative to you. Even though your car is zooming along the highway at 100 km/h (with respect to the road), your car is at rest relative to you. To get a result of zero for the velocity of your car relative to you, we subtract 100 km/h east (your velocity with respect to the ground) from the velocity of the car with respect to the ground. This method of subtracting your velocity with respect to the ground also works to find the velocity of something else (such as an oncoming car) with respect to you. Subtracting your velocity from the velocity of other objects is equivalent to adding the opposite of your velocity to these velocities. Figure 4.1 illustrates the process.

Most relative-velocity problems can be thought of as vector-addition problems. The trick is to keep track of which vectors we're adding (or subtracting). Let's explore this idea further.

EXPLORATION 4.1 – Crossing a river

You are crossing a river on a ferry. Assume there is no current in the river (that is, the water is at rest with respect to the shore). The ferry is traveling at a constant velocity of 7.0 m/s north with respect to the shore, while you are walking at a constant velocity of 3.0 m/s south relative to the ferry.

Step 1 - ***What is the ferry's velocity relative to you?*** We can use the notation $\vec{v}_{YF}$ to denote your velocity relative to the ferry, so we have $\vec{v}_{YF} = 3.0$ m/s south. The velocity of the ferry with respect to you is exactly the opposite of your velocity with respect to the ferry, so $\vec{v}_{FY} = 3.0$ m/s north. This relation is always true: the velocity of some object A with respect to another object B is the opposite of the velocity of B with respect to A ($\vec{v}_{AB} = -\vec{v}_{BA}$).

Step 2 - ***What is your velocity relative to the shore?*** If you stood still on the ferry, your velocity relative to the shore would match the ferry's velocity with respect to the shore. In this case, however, you are moving with respect to the ferry, so we have to add the relative velocities as vectors (see Figure 4.2). Your velocity with respect to the shore is your velocity relative to the ferry plus the ferry's velocity relative to the shore:

$$\vec{v}_{YS} = \vec{v}_{YF} + \vec{v}_{FS} = 3.0 \text{ m/s south} + 7.0 \text{ m/s north} = -3.0 \text{ m/s north} + 7.0 \text{ m/s north}.$$

$$\vec{v}_{YS} = 4.0 \text{ m/s north}.$$

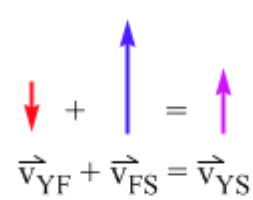

Figure 4.2: Finding your velocity relative to the shore by vector addition.

This vector-addition method is valid in general and works in more than one dimension.

A relative-velocity problem is a vector-addition problem. The velocity of an object A relative to an object C is the vector sum of the velocity of A relative to B plus the velocity of B relative to C:

$$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$$ (Equation 4.1: **The vector addition underlying relative velocity**)

Step 3 - ***Assume now that there is a current of 2.0 m/s directed south in the river, and that the ferry's velocity of 7.0 m/s north is relative to the water. What is the ferry's velocity relative to the shore? What is your velocity relative to the shore?***

To find the ferry's velocity with respect to the shore, we can use Equation 4.1 to combine the known values of the ferry's velocity with respect to the water and the water's velocity with respect to the shore.

In this case, using the subscript W to refer to the water, we get:

$$\vec{v}_{FS} = \vec{v}_{FW} + \vec{v}_{WS} = +7.0 \text{ m/s north} + 2.0 \text{ m/s south}.$$

$$\vec{v}_{FS} = +7.0 \text{ m/s north} - 2.0 \text{ m/s north} = +5.0 \text{ m/s north}.$$

Going against the current, the ferry takes longer to get to its destination, because it is moving slower with respect to the shore than when there was no current.

Now that we know the ferry's velocity with respect to the shore, we can apply Equation 4.1 to find your velocity with respect to the shore.

$$\vec{v}_{YS} = \vec{v}_{YF} + \vec{v}_{FS} = +3.0 \text{ m/s south + 5.0 m/s north.}$$

$$\vec{v}_{YS} = -3.0 \text{ m/s north + 5.0 m/s north = +2.0 m/s north.}$$

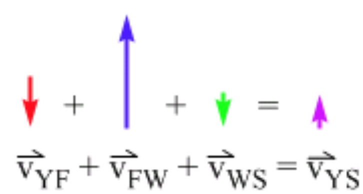

Figure 4.3: Finding your velocity relative to the shore by vector addition after adding a current in the river.

An alternate approach to finding your velocity with respect to the shore is to extend the procedure of Step 2 to three vectors, as represented in Figure 4.3. In this case, we get:

$$\vec{v}_{YS} = \vec{v}_{YF} + \vec{v}_{FW} + \vec{v}_{WS} = \text{3.0 m/s south + 7.0 m/s north + 2.0 m/s south.}$$

$$\vec{v}_{YS} = -3.0 \text{ m/s north + 7.0 m/s north – 2.0 m/s north = +2.0 m/s north.}$$

Key idea for relative velocity: Solving a relative velocity problem amounts to solving a vector-addition problem. In general, the velocity of an object A relative to an object C is the vector sum of the velocity of A relative to B plus the velocity of B relative to C: $\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$.

Related End-of-Chapter Exercises: 1, 5.

Essential Question 4.1 In Step 3 of Exploration 4.1, could you adjust your speed with respect to the ferry so that you are at rest with respect to the shore? If so, how would you adjust it?