Answer to Essential Question 3.8: The boxes will move together as one unit, so the boxes have the same acceleration. The net force acting on a box is the mass of the box multiplied by the acceleration. Thus, the green box, with the largest mass, experiences the largest net force.

3-9 Practicing the Method

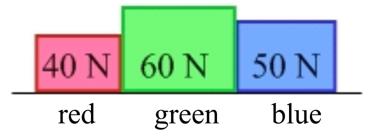

Figure 3.19: Three boxes placed side-by-side.

EXAMPLE 3.9 – Three boxes

Let's look in more detail at the system of three boxes that are side-by-side on a frictionless floor. The weights of the boxes are given in Figure 3.19. By exerting a constant force of $\vec{F} = 30$ N to the right on the red box, you cause the three boxes to accelerate. Use $g = 10$ m/s^2.

(a) What is the acceleration of the boxes?
(b) Using a notation in the form F_{RG}, which denotes the magnitude of the force that the red box applies to the green box, sketch free-body diagrams for (i) the red box; (ii) the green box; (iii) the system consisting of the red and green boxes together; and (iv) the system consisting of the green and blue boxes together.
(c) Which of the four free-body diagrams above would you use to determine F_{RG}, the magnitude of the force that the red box applies to the green box? What is F_{RG}?

SOLUTION

(a) Let's begin with a free-body diagram, but which system should we draw a free-body diagram for? Choosing the system carefully can make a problem easier to solve. The free-body diagrams for each box involve horizontal forces we don't yet know the magnitude of. Is there a free-body diagram we could draw so the only horizontal force acting is the 30 N force you apply?

As shown in Figure 3.20, the free-body diagram of the whole system involves only a downward force of gravity, an upward normal force, and the 30 N horizontal force. Let's choose a coordinate system with the positive x direction pointing to the right. Applying Newton's second law in that direction gives: $\sum \vec{F}_x = (m_R + m_G + m_B)\vec{a}_x$.

The only horizontal force acting on the combined system is your 30 N force in the $+x$ direction, so: $30\text{ N} = (m_R + m_G + m_B)\vec{a}_x$.

To find the masses, divide the weights by g, which we are taking here to be 10 m/s^2. The masses are $m_R = 4.0$ kg, $m_G = 6.0$ kg, and $m_B = 5.0$ kg. Solving for the acceleration gives

$$\vec{a}_x = \frac{+30\text{ N}}{(4.0\text{ kg} + 6.0\text{ kg} + 5.0\text{ kg})} = \frac{+30\text{ N}}{15.0\text{ kg}} = +2.0\text{ m/s}^2.$$

(b) The free-body diagram of the red box shows the 30 N horizontal force you exert, as well as the upward normal force exerted on the red box by the floor, $\vec{F}_{FR}$, and the downward force of gravity exerted on the red box by the Earth, $\vec{F}_{ER}$. The diagram must also account for the interaction between the green and red boxes – the green box exerts a force to the left on the red box, $\vec{F}_{GR}$. This force is smaller in magnitude than your 30 N force because the red box has a net force to the right.

The free-body diagrams of the green and red boxes are similar. In both cases, the upward normal force balances the downward force of gravity, while the fact that there is a net force to the right means that the force to the right is somewhat larger than the force to the left. Your 30 N force is applied only to the red box, so it appears only on the free-body diagram of the red box (or on the free-body diagram of a system involving the red box). The green box experiences a force to the right, from the red box, but it is less than 30 N.

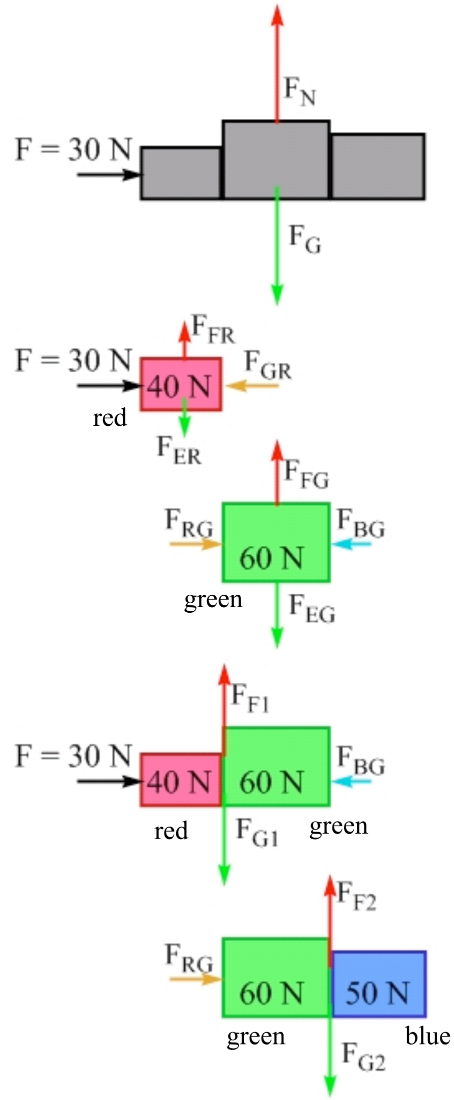

Figure 3.20: Various free-body diagrams for the three-box situation. In each case, the vertical forces cancel, so we can focus on the horizontal forces.

If we combine the red and green boxes into one system (system 1), the system's free-body diagram is the sum of the individual free-body diagrams. The net upward normal force $\vec{F}_{F1} = \vec{F}_{FR} + \vec{F}_{FG}$ balances the net downward force of gravity $\vec{F}_{E1} = \vec{F}_{ER} + \vec{F}_{EG}$. Horizontally, your 30 N force acts to the right, while the force $\vec{F}_{BG}$ that the blue box exerts on the green box is directed left.

We do not have to include $\vec{F}_{RG}$ and $\vec{F}_{GR}$ because, by Newton's third law, these are equal-and-opposite and thus cancel one another. Another reason to exclude this pair of forces is that they are internal forces in the system. Forces that belong on the free-body diagram come from external interactions, forces exerted on the system by things outside the system, such as by you, the floor, the Earth, and the blue box.

Combining the green and blue boxes into one system, system 2, the upward normal force from the floor $\vec{F}_{F2}$ balances the downward force of gravity $\vec{F}_{E2}$. Horizontally, there is only one horizontal force acting, the force the red box exerts on the green box $\vec{F}_{RG}$.

(c) To find $\vec{F}_{RG}$, we cannot use the system consisting of the red and green boxes together. $\vec{F}_{RG}$ is an internal force in that system, so it does not appear on the system's free-body diagram. $\vec{F}_{RG}$ appears on the free-body diagram of the green box, as well as on the free-body diagram of the system consisting of the green and blue boxes. In addition, $\vec{F}_{GR}$ appears on the free-body diagram of the red box so we could solve for that, since $\vec{F}_{RG}$ is equal in magnitude to $\vec{F}_{GR}$.

Let's use the last free-body diagram, because $\vec{F}_{RG}$ is the only horizontal force that appears on that diagram. Applying Newton's second law in the horizontal direction gives:

$$\sum \vec{F}_x = \left(m_G + m_B\right)\vec{a}_x \text{, so} \qquad \vec{F}_{RG} = (6.0 \text{ kg} + 5.0 \text{ kg}) * (+2.0 \text{ m/s}^2) = +22 \text{ N}.$$

Related End-of-Chapter Exercises: 36, 45, 58, 59.

Essential Question 3.9: Calculate the net force acting on each of the boxes in Example 3.9.