Answer to Essential Question 2.7: A common misconception is that the ball's acceleration is zero at the maximum-height point. In fact, the acceleration is $\vec{g}$, 9.8 m/s^2 down, during the entire trip.

This is what is shown on the acceleration graph, for one thing – the graph confirms that nothing special happens to the acceleration at t = 2.0 s, even though the ball is momentarily at rest. One reason for this is that the ball is under the influence of gravity the entire time.

2-8 Solving Constant-Acceleration Problems

Consider one more example of applying the general method for solving a constant-acceleration problem.

EXAMPLE 2.8 – Combining constant-acceleration motion and constant-velocity motion

A car and a bus are traveling along the same straight road in neighboring lanes. The car has a constant velocity of +25.0 m/s, and at $t = 0$ it is located 21 meters ahead of the bus. At time $t = 0$, the bus has a velocity of +5.0 m/s and an acceleration of +2.0 m/s^2.

When does the bus pass the car?

SOLUTION

1. **Picture the scene – draw a diagram.** The diagram in Figure 2.21 shows the initial situation, the positive direction, and the origin. Let's choose the positive direction to be the direction of travel, and the origin to be the initial position of the bus.

2. **Organize the data.** Data for the car and the bus is organized separately in Table 2.3, using subscripts C and B to represent the car and bus, respectively.

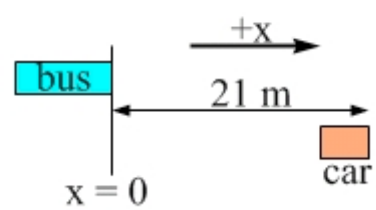

Figure 2.21: A diagram showing the initial positions of the car and the bus, the position of the origin, and the positive direction.

	Car	Bus
Initial position	$x_{iC} = +21\text{ m}$	$x_{iB} = 0$
Initial velocity	$v_{iC} = +25\text{ m/s}$	$v_{iB} = +5.0\text{ m/s}$
Acceleration	$a_C = 0$	$a_B = +2.0\text{ m/s}^2$

Table 2.3: Summarizing the information that was given about the car and the bus.

3. **Solve the problem.** Let's use Equation 2.8 to write expressions for the position of each vehicle as a function of time. Because we summarized all the data in Table 2.3 we can easily find the values of the variables in the equations.

For the car: $x_C = x_{iC} + v_{iC}\,t + \frac{1}{2}a_C\,t^2 = +21\text{ m} + (25\text{ m/s})t$.

For the bus: $x_B = x_{iB} + v_{iB}\,t + \frac{1}{2}a_B\,t^2 = 0 + (5.0\text{ m/s})t + (1.0\text{ m/s}^2)t^2$.

The bus passes the car when the vehicles have the same position. At what time does $x_C = x_B$? Set the two equations equal to one another and solve for this time (let's call it t_1).

$$+21\text{ m} + (25\text{ m/s})t_1 = (5.0\text{ m/s})t_1 + (1.0\text{ m/s}^2)t_1^2$$

Bringing everything to the left side gives: $(1.0\text{ m/s}^2)t_1^2 - (20\text{ m/s})t_1 - 21\text{ m} = 0$.

We can solve this with the quadratic equation, where $a = 1.0 \text{ m/s}^2$, $b = -20 \text{ m/s}$, and $c = -21 \text{ m}$.

$$t_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+(20 \text{ m/s}) \pm \sqrt{(400 \text{ m}^2/\text{s}^2) + (84 \text{ m}^2/\text{s}^2)}}{2.0 \text{ m/s}^2} = \frac{+(20 \text{ m/s}) \pm (22 \text{ m/s})}{2.0 \text{ m/s}^2}.$$

The equation gives us two solutions for t_1: Either $t_1 = +21$ s or $t_1 = -1.0$ s. Clearly the answer we want is +21 s. The other number does have a physical significance, however, so let's try to make some sense of it. The negative answer represents the time at which the car would have passed the bus if the motion conditions after $t = 0$ also applied to the period before $t = 0$.

4. **Think about the answer.** A nice way to check the answer is to plug $t_1 = +21$ s into the two position-versus-time equations from the previous step. If the time is correct the position of the car should equal the position of the bus. Both equations give positions of 546 m from the origin, giving us confidence that $t_1 = +21$ s is correct.

Note: This example is continued on the accompanying web site, solving for the time at which the car and the bus have the same velocity.

Related End-of-Chapter Exercises: 54 and 56.

Chapter Summary

Essential Idea: Describing motion in one dimension.
The motion of many objects (such as you, cars, and objects that are dropped) can be approximated very well using a constant-velocity or a constant-acceleration model.

Parameters used to Describe Motion
Displacement is a vector representing a change in position. If the initial position is $\vec{x}_i$ and the final position is $\vec{x}_f$ we can express the displacement as:

$$\Delta\vec{x} = \vec{x}_f - \vec{x}_i \;.$$ (Equation 2.1: **Displacement in one dimension**)

Average Velocity: a vector representing the average rate of change of position with respect to time. The MKS unit for velocity is m/s (meters per second).

$$\vec{\bar{v}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{\text{net displacement}}{\text{time interval}}.$$ (Equation 2.2: **Average velocity**)

While velocity is a vector, and thus has a direction, speed is a scalar.

Average Speed = $\bar{v} = \dfrac{\text{total distance covered}}{\text{time interval}}$ (Equation 2.3: **Average speed**)

Instantaneous Velocity: a vector representing the rate of change of position with respect to time at a particular instant in time. The MKS unit for velocity is m/s (meters per second).

$$\vec{v} = \frac{\Delta\vec{x}}{\Delta t},$$ (Equation 2.4: **Instantaneous velocity**)

where Δt is small enough that the velocity can be considered to be constant over that interval.

Instantaneous Speed: the magnitude of the instantaneous velocity.

Average Acceleration: a vector representing the average rate of change of velocity with respect to time. The MKS unit for acceleration is m/s^2.

$$\bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\text{change in velocity}}{\text{time interval}}.$$ (Equation 2.7: **Average acceleration**)

Instantaneous Acceleration: a vector representing the rate of change of velocity with respect to time at a particular instant in time.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t},$$ (Equation 2.8: **Instantaneous acceleration**)

where Δt is small enough that the acceleration can be considered constant over that interval.

The ***acceleration due to gravity,*** $\vec{g}$, is 9.8 m/s^2, directed down, at the surface of the Earth.

A General Method for Solving a 1-Dimensional Constant-Acceleration Problem

1. **Picture the scene.** Draw a diagram of the situation. Choose an origin to measure positions from, and a positive direction, and show these on the diagram.
2. **Organize what you know, and what you're looking for.** Making a table of data can be helpful. Record values of the variables used in the equations below.
3. **Solve the problem.** Think about which of the constant-acceleration equations to apply, and then set up and solve the problem. The three main equations are:

$$v = v_i + at.$$ (Equation 2.9: **Velocity for constant-acceleration motion**)

$$x = x_i + v_i t + \frac{1}{2}at^2.$$ (Equation 2.11: **Position for constant-acceleration motion**)

$$v^2 = v_i^2 + 2a\Delta x.$$ (Equation 2.12: **Connecting velocity, acceleration, and displacement**)

4. **Think about the answer(s)**. Check your answers, and/or see if they make sense.

Graphs

The velocity is the slope of the position-versus-time graph; the displacement is the area under the velocity-versus-time graph. The acceleration is the slope of the velocity-versus-time graph; the change in velocity is the area under the acceleration-versus-time graph.

Constant Velocity and Constant Acceleration

The motion of an object at rest is a special case of constant-velocity motion. In constant-velocity motion the position-versus-time graph is a straight line with a slope equal to the velocity.

Constant-velocity motion is a special case of constant-acceleration motion. In one dimension, if the acceleration is constant and non-zero the position-versus-time graph is quadratic while the velocity-versus-time graph is a straight line with a slope equal to the acceleration.