Answer to Essential Question 2.5: The position-versus-time graph is not a straight line, because the slope of such a graph is the velocity. The fact that the bus' velocity increases linearly with time means the slope of the position-versus-time graph also increases linearly with time. This actually describes a parabola, which we will investigate further in the next section.

2-6 Equations for Motion with Constant Acceleration

In many situations, we will analyze motion using a model in which we assume the acceleration to be constant. Let's derive some equations that we can apply in such situations. In general, at some initial time $t_i = 0$, the object has an initial position $\vec{x}_i$ and an initial velocity of $\vec{v}_i$, while at some (usually later) time *t,* the object's position is $\vec{x}$ and its velocity is $\vec{v}$.

Acceleration is related to velocity the same way velocity is related to position, so we can follow a procedure similar to that at the end of Section 2-3 to derive an equation for velocity.

Substitute $\vec{v}_f - \vec{v}_i$ for $\Delta\vec{v}$ in the rearrangement of Equation 2.8, $\Delta\vec{v} = \vec{a}\,\Delta t$.

This gives: $\vec{v}_f - \vec{v}_i = \vec{a}\,\Delta t = \vec{a}\left(t_f - t_i\right)$.

Generally, we define the initial time t_i to be zero: $\vec{v}_f - \vec{v}_i = \vec{a}\,t_f$.

Remove the "f" subscripts to make the equation as general as possible: $\vec{v} - \vec{v}_i = \vec{a}\,t$. We also generally remove the vector symbols, although we must be careful to include signs.

$v = v_i + a\,t$. (Equation 2.9: **Velocity for constant-acceleration motion**)

A second equation comes from the definition of average velocity (Equation 2.2):

Average velocity = $\bar{v} = \dfrac{\Delta\vec{x}}{\Delta t} = \dfrac{\vec{x} - \vec{x}_i}{t - t_i} = \dfrac{\vec{x} - \vec{x}_i}{t}$.

If the acceleration is constant the average velocity is simply the average of the initial and final velocities. This gives, after again dropping the vector symbols:

$\dfrac{v_i + v}{2} = \dfrac{x - x_i}{t}$. (Equation 2.10: **Connecting average velocity and displacement**)

Equation 2.10 is sometimes awkward to work with. If we substitute $v_i + at$ in for v (see Equation 2.9) in Equation 2.10, re-arranging produces an equation describing a parabola:

$x = x_i + v_i t + \dfrac{1}{2}at^2$. (Equation 2.11: **Position for constant-acceleration motion**)

We can derive another useful equation by combining equations 2.9 and 2.10 in a different way. Solving equation 2.9 for time, to get $t = \dfrac{v - v_i}{a}$, and substituting the right-hand side of that expression in for *t* in equation 2.10, gives, after some re-arrangement:

$v^2 = v_i^2 + 2a\,\Delta x$. (Eq. 2.12: **Connecting velocity, acceleration, and displacement**)

An important note about positive and negative signs.

When we make use of the equations on the previous page, we must be careful to include the appropriate positive or negative signs that are built into each of the variables. The first step is to choose a positive direction. If the initial velocity is in that direction, it goes into the equations with a positive sign. If the initial velocity is in the opposite direction (the negative direction), it goes into the equations with a negative sign. Apply a similar rule for the final velocity, the acceleration, the displacement, the initial position, and the final position. For all of those quantities, the sign is associated with the direction of the corresponding vector.

Motion with constant acceleration is an important concept. Let's summarize a general, systematic approach we can apply to situations involving motion with constant acceleration.

A General Method for Solving a One-Dimensional Constant-Acceleration Problem

1. **Picture the scene.** Draw a diagram of the situation. Choose an origin to measure positions from, and a positive direction, and show these on the diagram.
2. **Organize what you know, and what you're looking for.** Making a table of data can be helpful. Record values of the variables used in the equations below.
3. **Solve the problem.** Think about which of the constant-acceleration equations to apply, and then set up and solve the problem. The three main equations are:

$v = v_i + at$. (Equation 2.9: **Velocity for constant-acceleration motion**)

$x = x_i + v_i t + \frac{1}{2}at^2$. (Equation 2.11: **Position for constant-acceleration motion**)

$v^2 = v_i^2 + 2a\,\Delta x$. (Equation 2.12: **Connecting velocity, acceleration, and displacement**)

4. **Think about the answer(s)**. Check your answers to see if they make sense.

EXAMPLE 2.6 – Working with variables

In physics, being able to work with variables as well as numbers is an important skill. This can also produce insights that working with numbers does not. Let's say an object is dropped from rest from the top of a building of height *H*, while another object is dropped from rest from the top of a building of height 4*H*. Assuming both objects fall under the influence of gravity alone (that is, they have the same acceleration), compare the times it takes them to reach the ground.

SOLUTION

$v_i = 0$, because the objects are dropped from rest. Take the initial position to be the top of the building in each case, so $x_i = 0$. This reduces Equation 2.11 to $x = at^2/2$. Because the acceleration is the same in each case, the equation tells us the position is proportional to the square of the time. To quadruple the final position, as we are doing, we need to increase the time by a factor of two. The fall from the building that is four times as high takes twice as long. This is illustrated by the motion diagram in Figure 2.18.

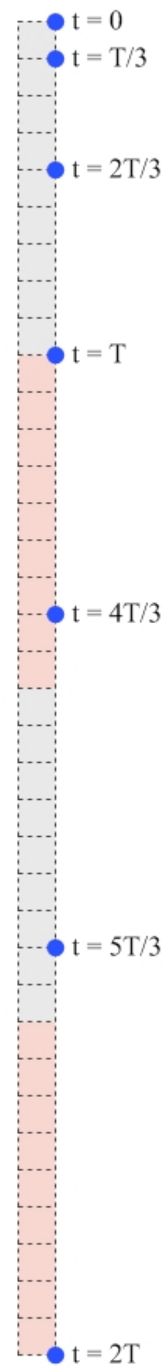

Figure 2.18: Falling from rest for double the time quadruples the distance traveled. The height of the diagram is 4*H*, four times the height of the smaller building in Example 2.6.

Related End-of-Chapter Exercises: 52 and 55

Essential Question 2.6: Return to the situation described in Example 2.6. Compare the velocities of the objects just before they hit the ground.