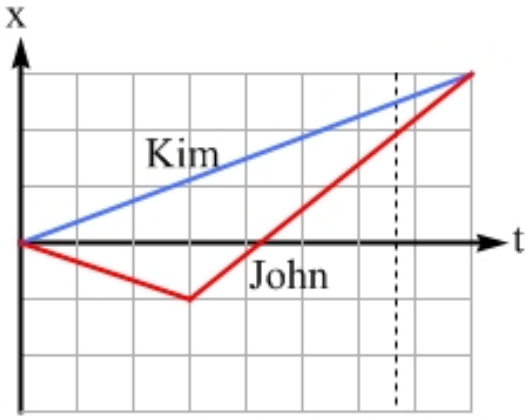
PROBLEM 1 – 10 points



The graph at right shows the position as a function of time for two students, Kim and John, as they move along a straight line.

For the time period shown in the graph …

[2 points] (a) which student travels a larger distance?

[ ] Kim [**X**] John [ ] equal for both

John covers an extra distance in the negative *x*-direction and then back to *x* = 0 that Kim does not cover.

[2 points] (b) which student's displacement has the larger magnitude?

[ ] Kim's [ ] John's [**X**] equal for both

They both have the same starting point and the same ending point.

[2 points] (c) which student has the larger average speed?

[ ] Kim [**X**] John [ ] equal for both

The average speed is the distance divided by the time. The time is the same, but John's distance is larger.

[2 points] (d) which student's average velocity has the larger magnitude?

[ ] Kim's [ ] John's [**X**] equal for both

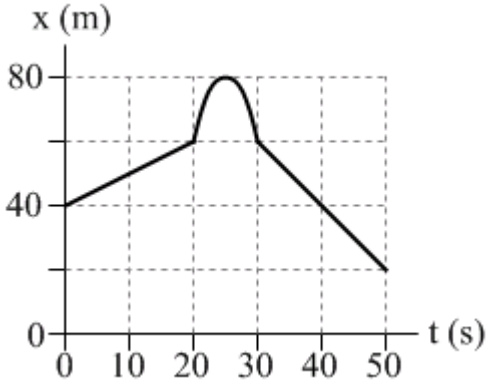
The average velocity is the displacement divided by the time. Both the displacement and the time are the same for Kim and John.

[2 points] (e) At the instant indicated by the dashed line on the graph, which student has the higher speed?

[ ] Kim [**X**] John [ ] equal for both

The speed is the magnitude of the slope of the position versus time graph, and John's graph has a larger slope at that instant.

PROBLEM 2 – 20 points



The graph shows your position as a function of time as you move along a sidewalk.

[4 points] (a) At t = 10 s, what is your:

Position: ___**50 m**__ Velocity: ___**1 m/s**__ Acceleration: ___**0 m/s^2**__

The position can be read from the graph. Remember that the slope of a position vs. time graph is the velocity. In our case, from t = 0 to t = 20 s the slope is constant and has the value 20 m / 20 s = 1 m/s (and is positive!). We also know that constant velocity implies zero acceleration.

[4 points] (b) At t = 40 s, what is your:

Position: ___**40 m**__ Velocity: ___**–2 m/s**__ Acceleration: ___**0 m/s^2**__

Same as before, but this time the slope is negative: $\Delta x/\Delta t$ = –40 m/ 20 s = –2 m/s. Again, from t = 30 s to t = 50 s the slope is constant, therefore the velocity is constant and hence there is no acceleration.

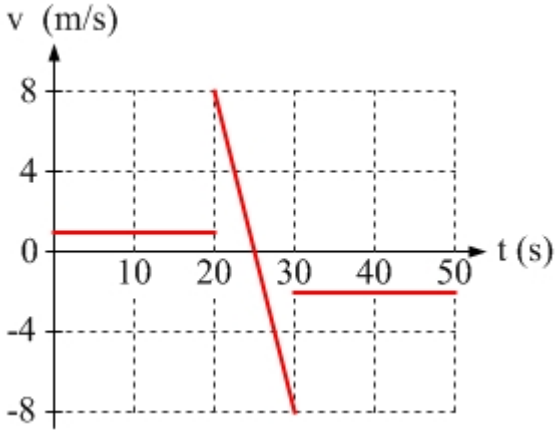
[4 points] (c) What is your average velocity over the interval from t = 0 s to t = 50 s?

Average velocity = displacement / time = $\Delta x/\Delta t$ = (x(t = 50 s) – x(t = 0 s)) / 50 s
= (20 m – 40 m) / 50 s = –20 m / 50 s = –0.4 m/s

[4 points] (d) What is your average speed over the interval from t = 0 s to t = 50 s?

Average speed = distance / time
To obtain the distance we must be careful to add all the distance walked while going forward AND backwards. From t = 0 s to t = 25 s we walked 80 m – 40 m = 40 m forward, and from that time on we walked backwards |20 m – 80 m| = 60 m. So the total distance is 100 m. So the average speed is 100 m / 50 s = 2 m/s

[4 points] (e) Sketch a rough graph of your velocity as a function of time over the interval from t = 0 s to t = 50 s.



PROBLEM 3 – 15 points

Two balls are launched at the same time. Ball A is released from rest from the top of a tall building of height H. Ball B is fired straight up from the ground with an initial velocity such that it just reaches the top of the same building. Neglect air resistance.

[3 points] (a) Which ball has the largest magnitude acceleration at the point they pass one another?

[ ] Ball A [ ] Ball B [**X**] neither, they're equal

Briefly justify your answer:
They're both in free fall, so the acceleration = g.

[3 points] (b) If ball A takes a time T to reach the ground, and ball B takes the same time T to reach the top of the building, which ball has the highest speed at time T/2?

[ ] Ball A [ ] Ball B [**X**] neither, they're equal

Briefly justify your answer:

Let's say ball B's initial velocity is v_i. Ball A's speed increases linearly with time from 0 to v_i while ball B's speed decreases linearly with time from v_i to 0. At T/2, both balls have a speed of v_i /2.

[4 points] How far from the ground are the two balls when they pass one another? Express your answer in terms of H.

There are several ways to approach this. One approach is to write down the equations of motion for the two balls:

Ball A: $y(t) = H - \frac{1}{2}gt^2$

Ball B: $y(t) = v_i t - \frac{1}{2}gt^2$

Set these equal and solve for the time, t′, when they're equal. This gives $H = v_i t'$ **.**

The time when the balls are at the same height is $t' = \frac{H}{v_i}$

What is v_i? Apply the equation $v^2 = v_i^2 + 2ay$ **to ball B, where at** $y = H$ **we have** $v = 0$**.**
This gives $0 = v_i^2 - 2gH$ **so** $v_i = \sqrt{2gH}$

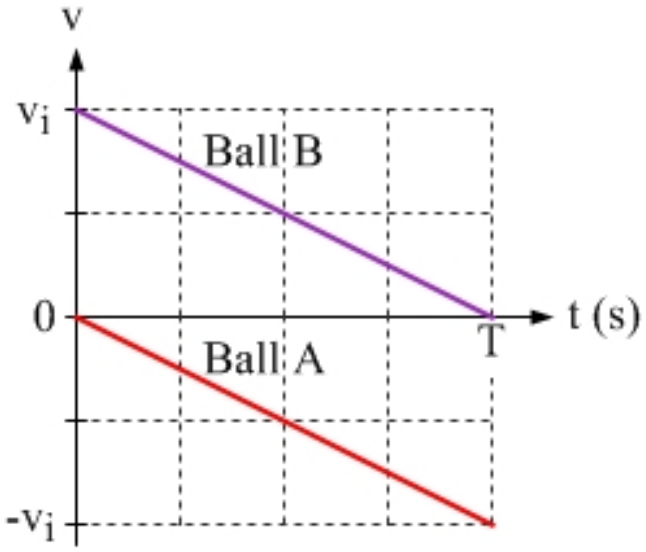
The time when the balls are at the same height is $t' = \frac{H}{\sqrt{2gH}}$

Plugging this into the ball A equation for position gives:

$$y(t) = H - \frac{1}{2}gt^2 = H - \frac{1}{2}g\frac{H^2}{2gH} = H - \frac{H}{4} = \frac{3H}{4}$$

If you recognized right away that the time when they're at the same height is T/2 you can get this answer more directly.

[5 points] (d) Sketch a graph showing the velocity of ball A, and the velocity of ball B, as a function from the time over the interval from when the balls are launched until ball A reaches the ground.



PROBLEM 4 – 15 points

A tortoise and a hare are having a 100 m race. When the starting gun goes off the hare lies down for a nap. The tortoise moves forward with a constant acceleration, reaching a speed of 2.0 m/s when she is 20 m from the starting line. After this, the tortoise travels at a constant velocity of 2.0 m/s until crossing the finish line. After 45 seconds the hare wakes up from his nap, and covers the 100 m with a constant acceleration of 2.0 m/s^2.

[6 points] (a) Who wins the race? Clearly justify your answer.

Determine the total time taken by each animal to reach the finish line.

The hare: Using $x = v_i t + \frac{1}{2}at^2$, with x = 100 m, $v_i = 0$, and a = 2 m/s^2 gives t = 10 s. Add this to the 45 s the hare was asleep for a total time of 55 seconds.

The tortoise: Using $v^2 = v_i^2 + 2ax$, with v = 2.0 m/s, $v_i = 0$, and x = 20 m gives an acceleration of a = 0.1 m/s^2.
Plug this into $v = v_i + at$ to find that the tortoise covers the first 20 m in 20 seconds.
The tortoise covers the remaining 80 m at a speed of 2 m/s. 80 m / 2 m/s = 40 seconds.
The tortoise's total time is 60 seconds for 100 m, slower than the hare.

The hare wins.

[2 points] (b) How much time passes between the winner reaching the finish line and the other animal reaching the finish line?

5 seconds (see above).

[2 points] (c) What is the distance between the animals when the winner crosses the finish line?

The tortoise is 5 seconds from the finish line when the hare reaches it. At a speed of 2.0 m/s

that's a distance of 2 m/s * 5 s = 10 meters.

[5 points] (d) What is the distance between the animals at the only time (other than at the instant the starting gun is fired) they have the same velocity?

While the tortoise is accelerating the hare is at rest, so the two have the same velocity when the hare reaches 2 m/s, the velocity of the tortoise the entire time the hare is moving.

Using $v = v_i + at$, with v = 2m/s, $v_i = 0$, and a = 2 m/s^2 gives t = 1 second. The hare reaches 2 m/s at a time of 1 second after he starts running, having covered a distance of $x = \frac{1}{2}at^2 = 1$ m.

In the meantime, the tortoise has been moving for 46 seconds. She covered 20 m in the first 20 seconds, and x = vt = 52 m in the remaining 26 seconds, for a total distance of 72 m. The distance between the animals at this point is 72 – 1 = 71 m.