Answer to Essential Question 1.5: $\vec{D} = -(6.77 \text{ m})\,\hat{x} + (5.59 \text{ m})\,\hat{y}$.

1-6 Coordinate Systems

Now that we have looked at an example of the component method of vector addition, in Example 1.5, we can summarize the steps to follow.

A General Method for Adding Vectors Using Components

1. Draw a diagram of the situation, placing the vectors tip-to-tail to show how they add geometrically.
2. Show the coordinate system on the diagram, in particular showing the positive direction(s).
3. Make a table showing the x and y components of each vector you are adding together.
4. In the last line of this table, find the components of the resultant vector by adding up the components of the individual vectors.

Coordinate systems

A coordinate system typically consists of an x-axis and a y-axis that, when combined, show an origin and the positive directions, as in Figure 1.10. A coordinate system can have just one axis, which would be appropriate for handling a situation involving motion along one line, and it can also have more than two axes if that is appropriate. An important part of dealing with vectors is to think about the coordinate system or systems that is/are appropriate for dealing with a particular situation. Let's explore this idea further.

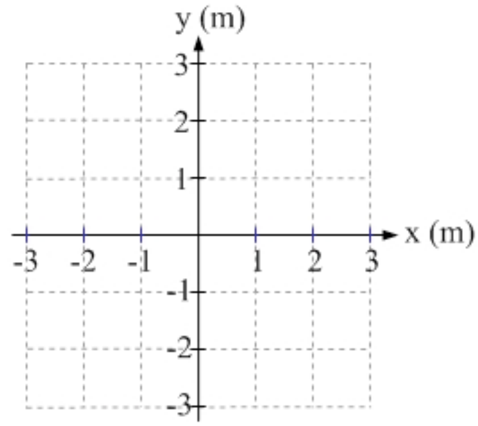

Figure 1.10: A typical x-y coordinate system.

EXPLORATION 1.6 – Buried treasure

While stranded on a desert island you find a note sealed inside a bottle that is half-buried near a big tree. Unfolding the note, you read: "Start 1 pace north of the big tree. Walk 10 paces northeast, 5 paces southeast, 6 paces southwest, 7 paces northwest, 4 paces southwest, and 2 paces southeast. Then dig." Realizing that your paces might differ in length from the paces of whoever left the note, rather than actually pacing out the distances you begin by drawing an x-y coordinate system in the sand, with positive x directed east and positive y directed north. After struggling to split the six vectors into components, however, you wonder whether there is a better way to solve the problem.

Step 1 - ***Is there only one correct coordinate system, or can you choose from a number of different coordinate systems to calculate a single resultant vector that represents the vector sum of the six vectors specified in the note?*** Any coordinate system will work, but there may be one coordinate system that makes the problem relatively easy, while others involve significantly more work to arrive at the answer. It's always a good idea to spend some time thinking about which coordinate system would make the problem easiest.

In fact, you should also think about whether the component method is even the easiest method to use to solve the problem. Adding vectors geometrically would also be a relatively easy way of solving this problem. Thinking about adding them geometrically (it might help to look at the six displacements, as sketched in Figure 1.11), in fact, leads us straight to the most appropriate coordinate system.

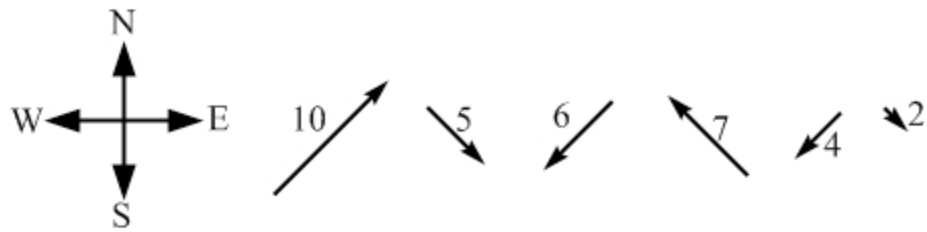

Figure 1.11: A sketch of the six displacements specified on the treasure map.

Step 2 - *What would be the simplest coordinate system to use to find the resultant vector?* One thing to notice is that the directions given are northeast, southeast, southwest, or northwest. An appropriate coordinate system is one that is aligned with these directions. For instance, we could point the positive x-direction northeast, and the positive y-direction northwest. In that case, out of the six different displacements, three are entirely in the x-direction and the other three are entirely in the y-direction. This makes the problem straightforward to solve. Figure 1.12 shows the vectors grouped by whether they are parallel to the x-axis or parallel to the y-axis.

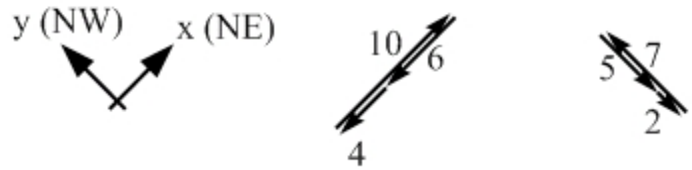

Figure 1.12: Choosing a coordinate system that fits the problem can make the problem easier to solve. In this case we have three vectors aligned with the x-axis and three vectors aligned with the y-axis.

Step 3 - *Where should you dig?* To determine where to dig, focus first on the displacements that are either in the $+x$ direction (10 paces northeast) or the $-x$ direction (6 paces southwest, and 4 paces southwest). Since the total of 10 paces southwest exactly cancels the 10 paces northeast, there is no net displacement along the x-axis.

Now turn to the y-axis, where we have 7 paces northwest (the $+y$ direction) and a total of $5 + 2 = 7$ paces southeast (the $-y$ direction). Once again these exactly cancel. Because the two components are zero, the resultant displacement vector has a magnitude of zero. You should dig at the starting point, 1 pace north of the tree (assuming you can figure out which way north is!). Digging at that spot, you find a box with a few car batteries, a 12-volt lantern, a solar cell, several wires, and a physics textbook. Reading through the book you figure out how to wire the solar cell to the batteries so the batteries are charged up while the sun shines, and you then figure out how to wire the batteries to the lantern to create a bright light you can use to signal passing planes. Using this system, you are rescued just a few days later, although you make sure to bury everything again carefully near the tree, and place the map back in the bottle, to help the next person who gets stranded there.

Key ideas for coordinate systems: Thinking carefully about the coordinate system to use can save a lot of work. Any coordinate system will work, but, in some cases, choosing the most appropriate coordinate system can make a problem considerably easier to solve.
Related End-of-Chapter Exercises: 5, 31, 41, 42.

Essential Question 1.6: In Exploration 1.6, the six displacements of 10 paces, 5 paces, 6 paces, 7 paces, 4 paces, and 2 paces happen to completely cancel one another because of their particular directions. If you could adjust the directions of each of the six vectors to whatever direction you wanted, what is the maximum distance they could take you away from the starting point?