Answer to Essential Question 1.4: Other examples of scalars include mass, distance, and speed. Examples of vectors, which have directions associated with them, include displacement, force, and acceleration.

1-5 Adding Vectors

EXAMPLE 1.5 – Adding vectors

Let's define a vector $\vec{C}$ as being the sum of the two vectors $\vec{A}$ and $\vec{B}$ from Exploration 1.4. A vector that results from the addition of two or more vectors is called a **resultant vector**.

(a) Draw the vectors $\vec{A}$ and $\vec{B}$ tip-to-tail to show geometrically the resultant vector $\vec{C}$.

(b) Use the components of vectors $\vec{A}$ and $\vec{B}$ to find the components of $\vec{C}$.

(c) Express $\vec{C}$ in unit-vector notation.

(d) Express $\vec{C}$ in terms of its magnitude and direction.

SOLUTION

(a) To add the vectors geometrically we can move the tail of $\vec{B}$ to the tip of $\vec{A}$, or the tail of $\vec{A}$ to the tip of $\vec{B}$. The order makes no difference. If we had more vectors, we could continue the process, drawing them tip-to-tail in sequence. The resultant vector always goes from the tail of the first vector to the tip of the last vector, as is shown in Figure 1.7.

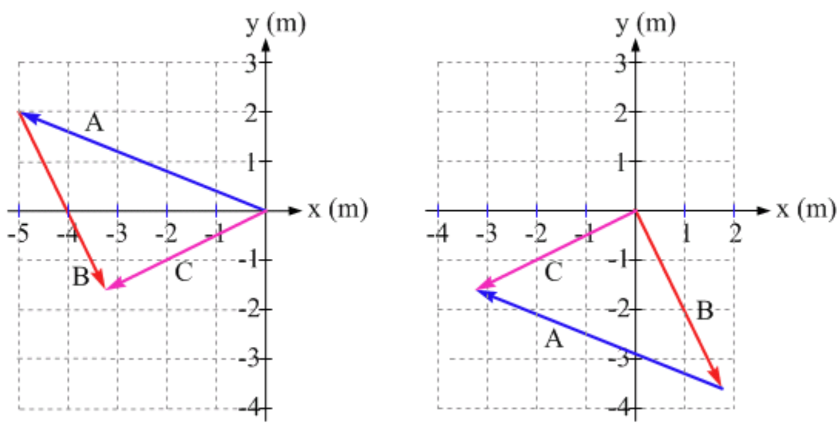

Figure 1.7: Adding vectors geometrically, tip-to-tail. In (a), the tail of vector $\vec{B}$ is placed at the tip of $\vec{A}$; in (b), the tail of vector $\vec{A}$ is placed at the tip of $\vec{B}$. The same resultant vector $\vec{C}$ is produced - the order does not matter.

(b) Now let's add the vectors using their components. We already know the x and y components of $\vec{A}$ and $\vec{B}$ (see Exploration 1.4), so we can use those to find the components of the resultant vector $\vec{C}$. Table 1.2 demonstrates the process. Note that the components of $\vec{A}$ are shown here to two decimal places, even though we know them with more precision. Because we'll be adding the components of $\vec{A}$ to the components of $\vec{B}$, which we know to two decimal places, our final answers should also be expressed with two decimal places.

Vector	*x*-component	*y*-component
$\vec{A}$	$\vec{A}_x = -(5.00\text{ m})\,\hat{x}$	$\vec{A}_y = +(2.00\text{ m})\,\hat{y}$
$\vec{B}$	$\vec{B}_x = (1.77\text{ m})\,\hat{x}$	$\vec{B}_y = -(3.59\text{ m})\,\hat{y}$
$\vec{C} = \vec{A} + \vec{B}$	$\vec{C}_x = \vec{A}_x + \vec{B}_x$ $\vec{C}_x = -(5.00\text{ m})\,\hat{x} + (1.77\text{ m})\,\hat{x}$ $\vec{C}_x = -(3.23\text{ m})\,\hat{x}$	$\vec{C}_y = \vec{A}_y + \vec{B}_y$ $\vec{C}_y = +(2.00\text{ m})\,\hat{y} - (3.59\text{ m})\,\hat{y}$ $\vec{C}_y = -(1.59\text{ m})\,\hat{y}$

Table 1.2: Adding the vectors $\vec{A}$ and $\vec{B}$ using components. The process is shown pictorially in Figure 1.8.

Note that we are solving this two-dimensional vector-addition problem by using a technique that is very common in physics – splitting a two-dimensional problem into two separate one-dimensional problems. It is very easy to add vectors in one dimension, because the vectors can be added like scalars with signs. To find $\vec{C}_x$, for instance, we simply add the *x*-components of $\vec{A}$ and $\vec{B}$ together. To find $\vec{C}_y$, we carry out a similar process, adding the *y*-components of $\vec{A}$ and $\vec{B}$. After finding the individual components of $\vec{C}$, we then combine them, as in parts (c) and (d) below, to specify the vector $\vec{C}$.

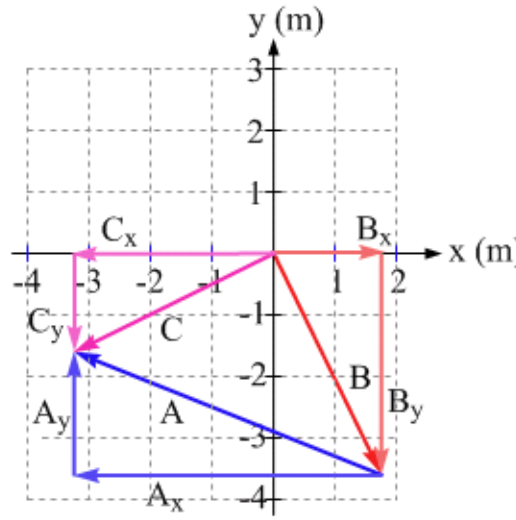

Figure 1.8: This figure illustrates the process of splitting the vectors into components when adding. Each component of the resultant vector, $\vec{C}$, is the vector sum of the corresponding components of the vectors $\vec{A}$ and $\vec{B}$.

(c) Using the bottom line in Table 1.2, the vector $\vec{C}$ can be expressed in unit-vector notation as:
$\vec{C} = \vec{C}_x + \vec{C}_y = -(3.23 \text{ m})\,\hat{x} - (1.59 \text{ m})\,\hat{y}$.

(d) If we know the components of a vector we can draw a right-angled triangle (see Figure 1.9) in which we know the lengths of two sides. Applying the Pythagorean theorem gives the length of the hypotenuse, which is the magnitude of the vector $\vec{C}$.

$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{3.23^2 + 1.59^2} = \sqrt{12.961} = 3.60 \text{ m}$$

To find the angle between $\vec{C}$ and $\vec{C}_x$ we can use the relationship:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{C_y}{C_x}.$$

This gives $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{1.59}{3.23}\right) = 26.2^\circ$.

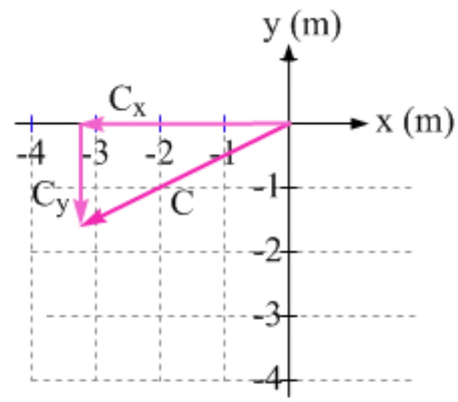

Figure 1.9: The components of the vector $\vec{C}$.

We have dropped the signs from the components, but, in stating the vector $\vec{C}$ correctly in magnitude-direction form, we can check the diagram to make sure we're accounting for which way $\vec{C}$ points: $\vec{C}$ = 3.60 m at an angle of 26.2° below the negative *x*-axis. The phrase "below the negative *x*-axis" accounts for the fact that the vector $\vec{C}$ has negative *x* and *y* components.

Related End-of-Chapter Exercises: 24 – 30.

Essential Question 1.5: Consider again the vectors $\vec{A}$ and $\vec{B}$ from Exploration 1.4 and Example 1.5. If the vector $\vec{D}$ is equal to $\vec{A} - \vec{B}$, express $\vec{D}$ in terms of its components.