Answer to Essential Question 1.3: The Pythagorean theorem is a special case of the Cosine Law that applies to right-angled triangles. With an angle of 90˚ opposite the hypotenuse, the last term in the Cosine Law disappears because cos(90˚) = 0, leaving $c^2 = a^2 + b^2$.

1-4 Vectors

It is always important to distinguish between a quantity that has only a magnitude, which we call a **scalar**, and a quantity that has both a magnitude and a direction, which we call a **vector**. When we work with scalars and vectors we handle minus signs quite differently. For instance, temperature is a scalar, and a temperature of +30˚C feels quite different to you than a temperature of –30˚C. On the other hand, velocity is a vector quantity. Driving at +30 m/s north feels much the same as driving at –30 m/s north (or, equivalently, +30 m/s south), assuming you're going forward in both cases, at least! In the two cases, the speed at which you're traveling is the same, it's just the direction that changes. So, ***a minus sign for a vector tells us something about the direction of the vector; it does not affect the magnitude (the size) of the vector***.

When we write out a vector we draw an arrow on top to represent the fact that it is a vector, for example $\vec{A}$. *A*, drawn without the arrow, represents the magnitude of the vector.

EXPLORATION 1.4 – Vector components

Consider the vectors $\vec{A}$ and $\vec{B}$ represented by the arrows in Figure 1.4 below. The vector $\vec{A}$ lines up exactly with one of the points on the grid. The vector $\vec{B}$ has a magnitude of 4.00 m and is directed at an angle of 63.8˚ below the positive *x*-axis. It is often useful (if we're adding the vectors together, for instance) to find the **components** of the vectors. In this Exploration, we'll use a two-dimensional coordinate system with the positive *x*-direction to the right and the positive *y*-direction up. Finding the *x* and *y* components of a vector involves determining how much of the vector is directed right or left, and how much is directed up or down, respectively.

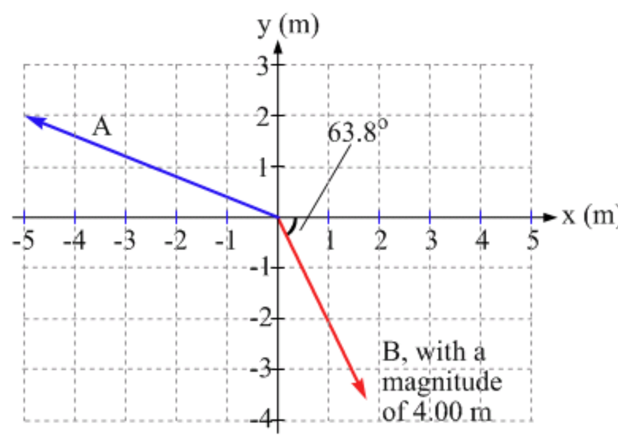

Figure 1.4: The vectors $\vec{A}$ and $\vec{B}$.

Step 1 - *Find the components of the vector* $\vec{A}$. The *x* and *y* components of $\vec{A}$ ($\vec{A}_x$ and $\vec{A}_y$, respectively) can be determined directly from Figure 1.4. Conveniently, the tip of $\vec{A}$ is located at an intersection of grid lines. In this case, we go exactly 5 m to the left and exactly 2 m up, so we can express the *x* and *y* components as:

$\vec{A}_x = +5 \text{ m}$ to the left, or $\vec{A}_x = -5 \text{ m}$ to the right.

$\vec{A}_y = +2 \text{ m}$ up.

This makes it look like we know the components of $\vec{A}$ to an accuracy of only one significant figure. The components are known far more precisely than that, because $\vec{A}$ lines up exactly with the grid lines. The components of $\vec{A}$ are shown in Figure 1.5.

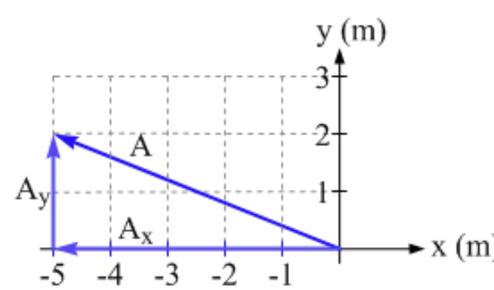

Figure 1.5: Components of the vector $\vec{A}$.

Step 2 – ***Express the vector $\vec{A}$ in unit-vector notation.*** Any vector is the vector sum of its components. For example, $\vec{A} = \vec{A}_x + \vec{A}_y$. This is shown graphically in Figure 1.5. It is rather long-winded to say $\vec{A} = -5$ m to the right + 2 m up. We can express the vector in a more compact form by using **unit vectors**. A unit vector is a vector with a magnitude of 1 unit. We will draw a unit vector with a carat (^) on top, rather than an arrow, such as $\hat{x}$. This notation looks a bit like a hat, so we say $\hat{x}$ as "x hat". Here we make use of the following unit vectors:

$\hat{x}$ = a vector with a magnitude of 1 unit pointing in the positive x-direction

$\hat{y}$ = a vector with a magnitude of 1 unit pointing in the positive y-direction

We can now express the vector $\vec{A}$ in the compact notation: $\vec{A} = (-5 \text{ m})\ \hat{x} + (2 \text{ m})\ \hat{y}$.

Step 3 - ***Find the components of the vector $\vec{B}$.*** We will handle the components of $\vec{B}$ differently from the method we used for $\vec{A}$, because $\vec{B}$ does not conveniently line up with the grid lines like $\vec{A}$ does. Although we could measure the components of $\vec{B}$ carefully off the diagram, we will instead use the trigonometry associated with right-angled triangles to calculate these components because we know the magnitude and direction of the vector.

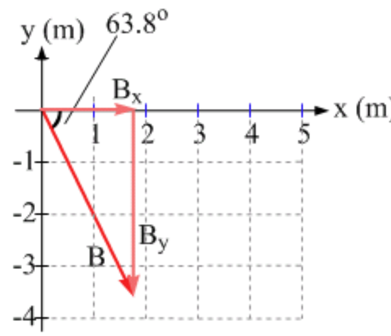

Figure 1.6: Components of the vector $\vec{B}$.

As shown in Figure 1.6, we draw a right-angled triangle with the vector as the hypotenuse, and with the other two sides parallel to the coordinate axes (horizontal and vertical, in this case). The x-component can be found from the relationship:

$$\cos\theta = \frac{B_x}{B}. \qquad \text{So } B_x = B\cos\theta = (4.00 \text{ m})\cos(63.8^\circ) = 1.77 \text{ m}.$$

We can use trigonometry to determine the magnitude of the component and then check the diagram to get the appropriate sign. From Figure 1.6, we see that the x-component of $\vec{B}$ points to the right, so it is in the positive x-direction. We can then express the x-component of $\vec{B}$ as:

$$\vec{B}_x = (+1.77 \text{ m})\ \hat{x}.$$

The y-component can be found in a similar way:

$$\sin\theta = \frac{B_y}{B}. \qquad \text{So, } B_y = B\sin\theta = 4.00\sin(63.8^\circ) = 3.59 \text{ m}.$$

The y-component of $\vec{B}$ points down, so it is in the negative y-direction. Thus:

$$\vec{B}_y = -(3.59 \text{ m})\ \hat{y}.$$

The vector $\vec{B}$ can now be expressed in unit-vector notation as:

$$\vec{B} = \vec{B}_x + \vec{B}_y = (1.77 \text{ m})\hat{x} - (3.59 \text{ m})\hat{y}.$$

Key ideas for vectors: It can be useful to express a vector in terms of its components. One convenient way to do this is to make use of unit vectors; a unit vector is a vector with a magnitude of 1 unit. **Related End-of-Chapter Exercises: 6, 18.**

Essential Question 1.4: Temperature is a good example of a scalar, while velocity is a good example of a vector. List two more examples of scalars, and two more examples of vectors.