Answer to Essential Question 1.2: There are only two significant figures in the value 0.0035 m, because it can be written as 3.5 x 10^{-3} m, which has only two significant figures. There are four significant figures in the value 35.00 m, because it can be written as 3.500 x 10^{1} m, which has four significant figures. Trailing zeroes are very important! 3.5 x 10^{1} m and 3.500 x 10^{1} m represent the same length, but in the second case we know the length with greater precision than we do in the first case.

1-3 Trigonometry, Algebra, and Dimensional Analysis

Solving a physics problem often involves the geometry of right-angled triangles. Such a triangle is shown in Figure 1.2. In a right-angled triangle there are several relationships between the angle shown in the diagram and the different sides of the triangle, including:

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{a}{c}; \quad \cos\theta = \frac{adjacent}{hypotenuse} = \frac{b}{c}; \quad \tan\theta = \frac{opposite}{adjacent} = \frac{a}{b}.$$

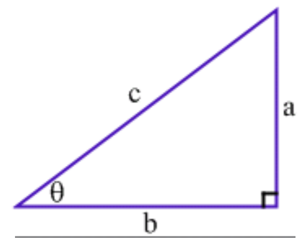

Figure 1.2: A right-angled triangle.

In a right-angled triangle, the Pythagorean Theorem relates the three sides:

$$c^2 = a^2 + b^2.$$ (Equation 1.1: **The Pythagorean theorem**).

A few special right-angled triangles include:
- the 3-4-5 triangle in which the sides are in a 3:4:5 ratio.
- The 5-12-13 triangle in which the sides are in a 5:12:13 ratio.
- The 30˚-60˚-90˚ triangle in which the sides are in a 1: $\sqrt{3}$:2 ratio.

Many triangles do not have a 90˚ angle. For a general triangle, such as that in Figure 1.3, if we know the length of two sides and one angle, or the length of one side and two angles, we can use the Sine Law and the Cosine Law to find the other sides and angles.

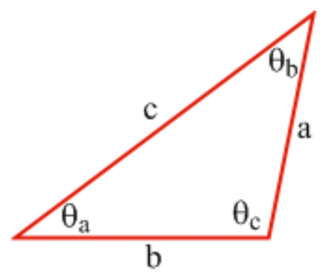

Figure 1.3: A general triangle.

$$\frac{\sin\theta_a}{a} = \frac{\sin\theta_b}{b} = \frac{\sin\theta_c}{c}.$$ (Equation 1.2: **Sine Law**)

$$c^2 = a^2 + b^2 - 2ab\cos\theta_c.$$ (Equation 1.3: **Cosine Law**)

Algebra

In addition to understanding what concepts to apply in solving a particular physics problem, you will need to know how to manipulate equations to solve for a particular unknown. In other words, you'll need to do algebra.

EXAMPLE 1.3A – Solving an equation using algebra

Solve for *v* in the following equation: $4v^2 - 7 = 3 - v^2$. Take a minute to solve the equation on your own before looking at the solution.

SOLUTION

To solve for a particular variable, you generally isolate that variable on one side and place everything else on the other side. Taking a step-by-step approach gives:

1. Bring all v terms to the left by adding v^2 to both sides: $5v^2 - 7 = 3$
2. Isolate the v term on the left by adding 7 to both sides: $5v^2 = 10$
3. Divide by 5: $v^2 = 2$
4. Solve for v: $v = \pm\sqrt{2}$

It is tempting to say that $v = \sqrt{2}$, but it is important to remember that the negative square root is also a possibility.

We did not concern ourselves with units above, but whenever you come up with an equation it is a good idea to do some dimensional analysis (that is, check your units). If the units check out, that does not necessarily mean your equation is correct. If your units do not check out, however, you know for sure there is something wrong with the equation.

EXAMPLE 1.3B – Using dimensional analysis

You're trying to solve for the velocity of a ball, 3 seconds after you throw it straight up in the air. You know that the velocity v has units of m/s, and you know the following parameters (defining the positive direction to be up): the initial velocity is $v_i = 20 \text{ m/s}$; the acceleration is $a = -10 \text{ m/s}^2$; and the time is $t = 3$ s. Your friend Sara says the equation connecting these variables is: $v = v_i + at/2$. Your friend Bob claims the equation is: $v = v_i + at^2$. Can dimensional analysis (checking the units) help you to rule out one or both of these equations as incorrect?

SOLUTION

Let's try both equations, keeping careful track of the units as we go.

Sara's method: $v = v_i + \frac{1}{2}at = 20\frac{\text{m}}{\text{s}} + \frac{1}{2}\left(-10\frac{\text{m}}{\text{s}^2}\right)(3 \text{ s}) = 20\frac{\text{m}}{\text{s}} - 15\frac{\text{m}}{\text{s}}$.

For Sara's equation, the left-hand side (v) has units of m/s, and both terms on the right-hand side also have units of m/s. This is good. ***Quantities that are added or subtracted must have the same units***, and ***the units on one side of an equation must match the units on the other***.

Bob's method: $v = v_i + at^2 = 20\frac{\text{m}}{\text{s}} + \left(-10\frac{\text{m}}{\text{s}^2}\right)(3 \text{ s})^2 = 20\frac{\text{m}}{\text{s}} - 90 \text{ m}$.

Bob's equation is incorrect, because the two terms on the right do not have the same units, and the units of the last term do not match the units of the left side of the equation.

In fact, *neither Sara nor Bob has the correct equation*. As we will see in chapter 2, the correct equation relating the velocity to the initial velocity, acceleration, and time is $v = v_i + at$. Dimensional analysis let us know that Bob's equation was incorrect, but it could not tell us that Sara's equation had an extra factor of ½ in one term, because that extra factor had no units associated with it. Dimensional analysis can be helpful, but it is just one tool in our problem-solving toolkit, and it needs to be used appropriately.

Related End-of-Chapter Exercises: 38, 45.

Essential Question 1.3: What is the connection between the Pythagorean theorem and the Cosine law?