Energy of a Rolling Object

INTRODUCTION

In this experiment, we will apply the Law of Conservation of Energy to objects rolling down a ramp. As an object rolls down the incline, its gravitational potential energy is converted into both translational and rotational kinetic energy. The translational kinetic energy is

$$KE_{\text{trans}} = (1/2)mv^2 \quad (1)$$

whereas the rotational kinetic energy is

$$KE_{\text{rot}} = (1/2)I\omega^2 \quad (2)$$

In this last equation ω is the angular velocity in radians/sec, and I is the object's moment of inertia. For objects with simple circular symmetry (e.g. spheres and cylinders) about the rotational axis, I may be written in the form:

$$I = kmr^2 \quad (3)$$

where m is the mass of the object and r is its radius. The geometric factor k is a constant which depends on the shape of the object:

$\boldsymbol{k = 2/5 = 0.4}$	for a uniform solid sphere,
$\boldsymbol{k = 1/2 = 0.5}$	for a uniform disk or solid cylinder,
$\boldsymbol{k = 1}$	for a hoop or hollow cylinder.

If the object rolls without slipping, then the object's linear velocity and angular speed are related by $v = r\omega$. Substituting equation 3 and the expression for v into equation 2, we obtain:

$$KE_{\text{rot}} = (1/2)kmv^2 \quad (4)$$

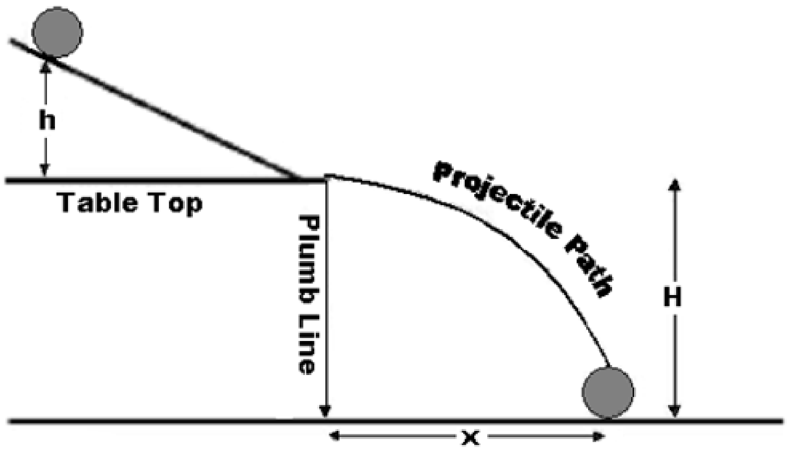

Figure 1

Consider a round object rolling down a ramp as in the illustration above. Assuming no loss of energy we may write the conservation of energy equation as:

$$\textbf{total energy at top of ramp} = \textbf{total energy at bottom of ramp,}$$
$$\mathbf{E_{gravitational}} = \mathbf{E_{translational}} + \mathbf{E_{rotational}}$$

or,

$$mgh = (1/2)mv^2 + (1/2)kmv^2. \qquad (5)$$

We can determine v by analyzing the motion of the ball after it leaves the table. Recalling that the horizontal and vertical motion of a projectile may be treated independently we have,

$$x = vt \qquad (6a)$$

and

$$H = (1/2)gt^2 \qquad (6b)$$

where t is the time of flight, x is the horizontal range, and H is the vertical height of the ramp above the floor. These two equations (6a and 6b) can be combined, eliminating t, to obtain the following expression for the velocity in terms of x and H.

$$v^2 = gx^2/2H \qquad (7)$$

Therefore, the energy of the rolling object can be analyzed entirely in terms of the measured values: m, h, H, x, and the acceleration due to gravity, g.

PROCEDURE

Part 1

1 Level the bottom surface of the ramp and the tray.

2 Choose one object from the collection of solid spheres, solid cylinders, and circular rings provided. Identify the size and weight of this object by recording its diameter and mass.

3 Roll the object down the ramp, starting from the **top** of the ramp, noticing at what point the object lands in the catch tray. Adjust the tray so that this point is towards the far end of the tray.

4 A sheet of white paper taped in the tray and overlaid by a piece of carbon paper ("carbonized" face down) will record the impact point of the object. *Use only one sheet of carbon paper and do not tape it down, move it around to record the impacts!* From the impact point you can determine x. Choose at least **6 heights**, $\boldsymbol{h}$, from 15 cm to 35 cm and roll the object down the ramp **several times** from each height. State very specifically how you determine the height h. Try to catch the object after the first bounce to avoid stray marks on the paper.

5 Measure the horizontal distance from the edge of the table to each mark, and enter this distance as x in Table 1. Plumb bobs will help you find the right point from which to measure (your instructor will demonstrate this). It's a good idea to check periodically to make sure that your tray has not shifted under the impact of the objects, which would introduce error.

6 Take several measurements of H, the distance the object falls from the bottom of the ramp.

Part 2

Select 5 different objects and record descriptive information about the physical characteristics of each object (shape, mass, diameter, etc.). Before you roll each object down the ramp, predict and record the relative horizontal distance that each will travel (rank order). Roll each object several times from the same initial height to observe the differences in the horizontal distances each lands from the end of the ramp. Be careful to use a procedure to ensure that objects of different radii roll through the same vertical distance, h, and explain the method you used to accomplish this task. Measure and record the average x for each object.

Be sure to initial your data, have your TA initial your data, and hand in a copy before you leave the lab room.

ANALYSIS

Part 1

1 Complete the data tables with the information needed to compute the initial potential energy and the translational and rotational kinetic energies for each release height. Calculate these energy values and determine the percentage of energy "lost" to non-conservative factors that were assumed to be negligible. What affect does the release height have on your results?

2 Estimate the uncertainty in your measurements and compare with the fractional loss of energy. Based on this comparison, was mechanical energy conserved?

Part 2

Did the different objects behave according to your predictions? What one factor is most important in determining the horizontal distance traveled? Use your results to find a general rule for predicting the efficiency in converting gravitational energy to translational kinetic energy for an object rolling down a ramp.

DISCUSSION

To what extent was energy conserved for this experiment? What non-conservative factors are most responsible for the loss of mechanical energy in this experiment? How were the results affected by the initial release height or the type of object? Is there a general rule for determining the percentage of initial potential energy that is converted to translational kinetic energy for an object rolling down a ramp? What one factor is most important in determining this energy conversion ratio? What factors do not matter? How does friction affect this experiment? If there is not enough friction for the object to roll without slipping, how does this affect the horizontal distance, x? What else did you learn from this experiment?

