The Pendulum

OBJECTIVES

- to verify Galileo's observations that the period of a pendulum is independent of the mass and angular amplitude (for small angles)

- to determine how the period of a pendulum depends upon the length of the string

- to calculate the experimental mass of Earth

EQUIPMENT

set of 6 identical washers that will act as the simple pendulum

support stand with a photogate connected to the Science Workshop interface

meter stick

protractor

INTRODUCTION AND THEORY

A simple pendulum consists of a point mass and a string. The point mass is suspended at one end of the string and the other end of the string is held fixed. The motion of a pendulum takes place in a vertical plane as illustrated in fig. 1, where the angle θ is measured from the vertical line to the string.

The gravitational force exerted upon the mass m can be decomposed into two perpendicular components. The sum of the forces in the direction along the string is

$$T_s - mg \, \cos\theta = ma_r, \tag{1}$$

where $a_r = v^2/r$ is the radial acceleration. In the other direction, the sum of the forces is

$$-mg \, \sin \, \theta = ma_t, \tag{2}$$

where a_t is the tangential acceleration and is equal to $-g \, \sin \, \theta$. This acceleration is tangent to the path of the pendulum at every point.
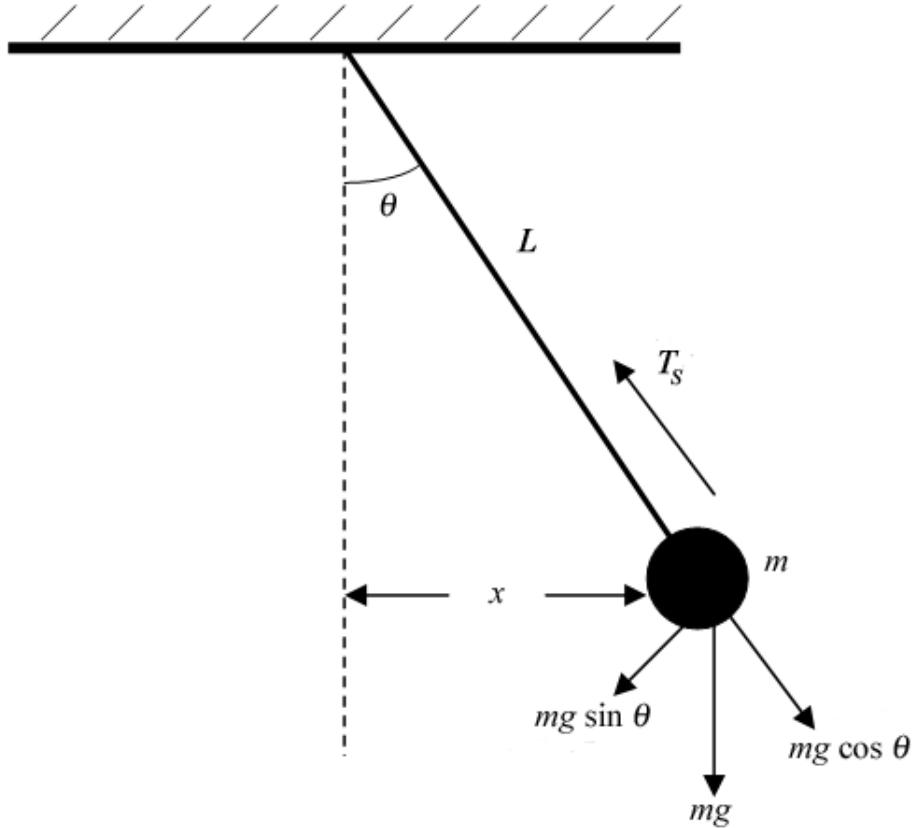
Figure 1: The force diagram for a pendulum.

If the angle θ is sufficiently small, then $\sin\theta \approx \tan\theta \approx x/L$, and eqn. 2 becomes

$$F_{\text{net}} = -mg\,\frac{x}{L} = -\left(\frac{mg}{L}\right)x. \tag{3}$$

Eqn. 3 is the equation for simple harmonic motion and from this the period of the pendulum can be determined to be

$$T = 2\pi\sqrt{\frac{L}{g}}. \tag{4}$$

The motion of the pendulum is periodic in time, which is the basis for pendulum clocks.

If the angle θ is not small, $\sin\theta$ has to be expanded into an infinite series, and the result for the period becomes

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{4}\,\sin^2\frac{\theta}{2} + \frac{9}{64}\,\sin^4\frac{\theta}{2} + \dots\right). \tag{5}$$

Note that eqn. 5 can be used to estimate the error in the period when θ is not small. For example, when $\theta = 10°$,

$$\frac{\Delta T}{T} = \frac{1}{4}\ \sin^2\ \frac{\theta}{2} + \frac{9}{64}\ \sin^4\ \frac{\theta}{2} = 1.907 \times 10^{-3}, \tag{6}$$

which is approximately a 0.2% error. Additional information about the pendulum can be found in your textbook.

PROCEDURE

Please print the worksheet for this lab. You will need this sheet to record your data.

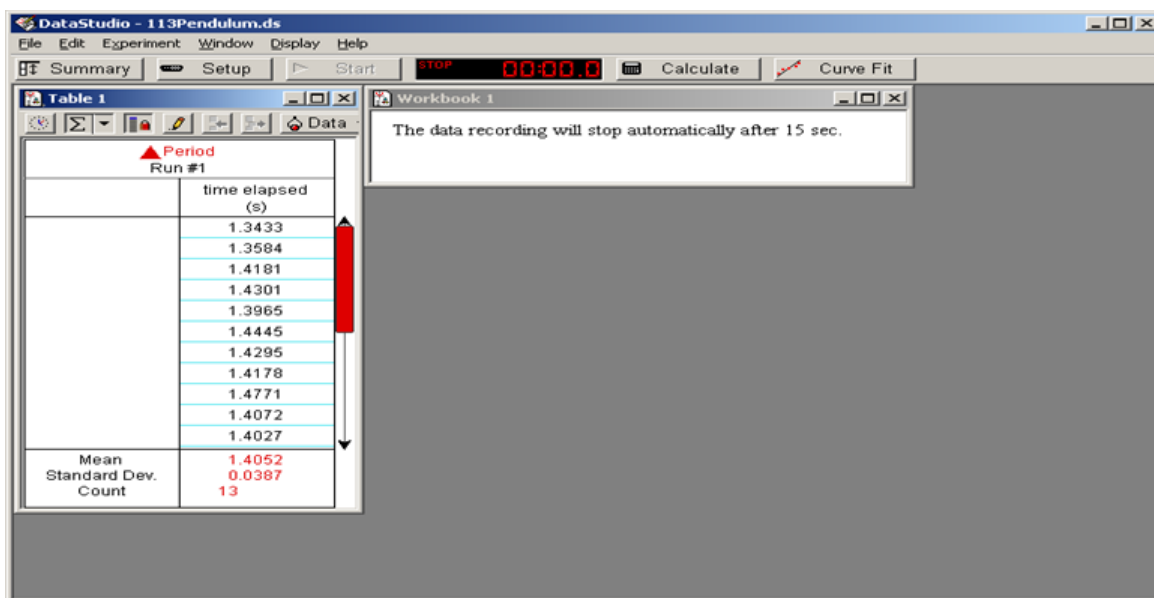Open the pre-set experiment file: pirt-PHY 113-PreSetUpLabs-Pendulum.

Figure 2: A sample of the DataStudio® file for the pendulum exercise.

We will use a photogate timer to make measurements of the period T of a simple pendulum. A photogate consists of an infrared diode, called the source diode, which emits an invisible infrared light beam. This beam is detected by another diode, the detector diode. When the mass, m, passes between the source and the detector, the infrared light beam is interrupted, producing an electronic signal that is used to start or stop an electronic timer.

The photogate is set up to measure the period of oscillations of the pendulum, i.e. the time it takes for the pendulum to go all the way out and all the way back. In other words, the pendulum has to pass through the gate twice for one full swing. The photogate measures the time between the consecutive interruptions of its beam. See fig. 3.
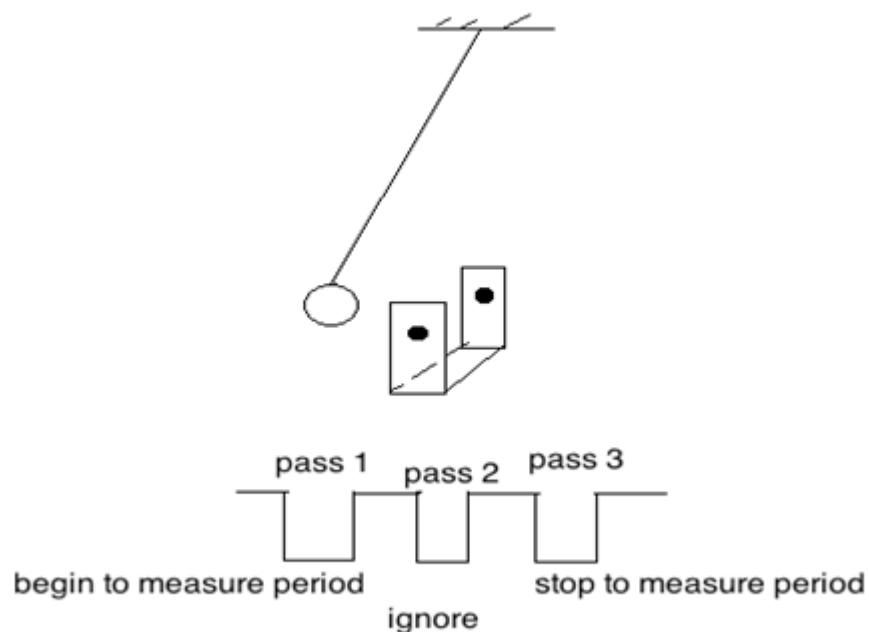
Figure 3

Check that the timer is working properly by passing a pencil three times through the photogate. The timer should start on the first pass, ignore the second pass, and stop on the third pass.

Be aware that the photogate can get confused if the string crosses the beam or if the beam passes through the opening in a washer. Use masking tape to close the opening in a washer. Always keep an eye on the pendulum while it is swinging so that it does not drift into a collision with the delicate and expensive photogate.

Experiment I: Measure the period of oscillations for pendulum bobs of different masses while keeping the length of the pendulum and amplitude fixed.

Allow the mass to swing back and forth a couple of times before you start making measurements to let the pendulum "settle". To start the data acquisition, press the "Start" button on the tool bar menu. Use small amplitudes (angles) of about 5°.

1 Put one washer at the end of the string. In Graphical Analysis, create columns to record the mean times (in seconds) and the number of the washers. All of the washers have the same mass, so the numbers of washers is proportional to the mass of the pendulum bob. START the timer in DataStudio®. It will stop automatically in 15 s. Record the mean value shown in DataStudio® of the period in the GA.

2 Continue changing the mass of the pendulum by adding one washer at a time. Measure the period of oscillations for each mass. Keep recording the mean value of the period and the number of the washers in GA.

3 In Graphical Analysis, make a graph of the pendulum period (the mean value) vs. the mass (the number of washers). All of the washers have the same mass, so the numbers of washers are proportional to the mass of the pendulum bob.

4 Record the slope and its uncertainty of the period vs mass graph in the Lab 7 worksheet.

Experiment II: Measure the period of oscillations for pendulum bobs at different amplitudes while keeping length and mass fixed.

1 In Graphical Analysis, create columns to record the mean value of the period and the starting amplitudes (angles). Place two washers on the string. START the timer in DataStudio®. It will stop automatically in 15 s. Record the mean value of the period in the GA.

2 Repeat measurements of the period of oscillations for angles 5°, 10°, 15°, 20°, 30°, and 40°.

3 Discuss the relationship between period and amplitude in the discussion section. Address in your discussion if the period stayed constant for all the angles.

Experiment III: Measure the period of oscillations for pendulum bobs while varying length.

Keep the initial angle (e.g. 10°) and mass (two-washers) constant. The length of a simple pendulum is the distance from the pivot point to the center of mass of the pendulum bob.

1 Measure the period of oscillations for 6 different lengths of the pendulum.

2 Enter your measured values of the period T (the mean value) and corresponding length L into GA. Create a new column for the value of T^2 (DATA-NEW CALCULATED COLUMN). Plot T^2 vs. L and apply linear fit from the Analyze pull down menu. The slope of this graph and its error will allow you to determine the acceleration due to gravity (g) and the error. Record the slope of the graph and its error in the Lab 7 worksheet.

From Equation 4:

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{7a}$$

$$T^2 = 4\pi^2\,\frac{L}{g} \tag{7b}$$

$$T^2 = \frac{4\pi^2}{g}\ \cdot\ L = \text{slope}\ \cdot\ L \tag{7c}$$

$$\text{slope} = \frac{4\pi^2}{g}. \tag{7d}$$

---

3 The slope and its uncertainty are calculated by the computer. Use the rules for error propagation to determine the uncertainty in g. The relative error for the calculated gravitational acceleration can be found as:

$$\frac{\Delta g}{g} = \frac{\Delta \text{slope}}{\text{slope}}.$$
(8)

4 Report your results of the gravitational acceleration and its error in the result section.

Calculating the mass of Earth

Since the factor of g occurs in the period of a pendulum, it is possible to use the measurements taken above to determine the mass of the Earth. The acceleration due to gravity near the surface of the Earth is defined as

$$g = \frac{GM_{\text{E}}}{r_{\text{E}}^2}.$$
(9)

Since G, M_E and r_E have fixed values, all objects near the surface of the Earth, regardless of their mass, are subject to the same acceleration.

Substituting this expression for g in eqn. 4 for the period of the pendulum we get:

$$T = 2\pi \sqrt{\frac{Lr_{\text{E}}^2}{GM_{\text{E}}}}.$$
(10)

For one of your trials above, use the measured length of the pendulum and the average period as determined by DataStudio® to compute the mass of the Earth. Compare it with the accepted value of 5.97×10^{24} kg by calculating the relative discrepancy.