Uncertainty Help

A summary of rules for calculations with numbers that include uncertainty

A measured value is expressed like this:

$$\boldsymbol{x \pm \delta x},$$

where $\boldsymbol{x}$ is the **measured value** (what we think the measurement is) and $\boldsymbol{\delta x}$ is the **absolute uncertainty** (how much we think we could be off by).

Note that $\boldsymbol{\delta}$ is the lower-case Greek "delta". Some texts use the upper case delta $\boldsymbol{\Delta}$.

We could also express this measurement as a maximum and minimum value

$$\boldsymbol{x_{\max} = x + \delta x}$$

and

$$\boldsymbol{x_{\min} = x - \delta x}.$$

If you know the maximum and minimum values, $\boldsymbol{x_{\max}}$ and $\boldsymbol{x_{\min}}$, and you want to express the value in $\boldsymbol{x \pm \delta x}$ form,

$$\boldsymbol{x = \frac{(x_{\max} + x_{\min})}{2}}$$

and

$$\boldsymbol{\delta x = \frac{(x_{\max} - x_{\min})}{2}}.$$

Relative uncertainty is

$$\textbf{relative uncertainty as a percentage} = \boldsymbol{\frac{\delta x}{x} \times 100}.$$

To find the absolute uncertainty if we know the relative uncertainty,

$$\textbf{absolute uncertainty} = \frac{\textbf{relative uncertainty}}{\textbf{100}} \times \textbf{measured value}.$$

Calculations using numbers with uncertainty

Consider two numbers that have uncertainty $\boldsymbol{x \pm \delta x}$ and $\boldsymbol{y \pm \delta y}$.

Addition: Add the absolute uncertainty of the original numbers to find the absolute uncertainty of the sum.

$$\boldsymbol{(x \pm \delta x) + (y \pm \delta y) = (x + y) \pm (\delta x + \delta y)}$$

Subtraction: Add the absolute uncertainty of the original numbers to find the absolute uncertainty of the difference.

$$(x \pm \delta x) - (y \pm \delta y) = (x - y) \pm (\delta x + \delta y)$$

Multiplication: Add the relative uncertainty of the original numbers to find the relative uncertainty of the product.

$$\textbf{relative uncertainty of } x \times y = \textbf{relative uncertainty of } x + \textbf{relative uncertainty of } y$$

Division: Add the relative uncertainty of the original numbers to find the relative uncertainty of the quotient.

$$\textbf{relative uncertainty of } x \div y = \textbf{relative uncertainty of } x + \textbf{relative uncertainty of } y$$

Raising to a power: When we raise a number with uncertainty to a power n, the relative uncertainty of the result is n times the relative uncertainty of the original number.

$$\textbf{relative uncertainty } of\ x^n = n \times \textbf{relative uncertainty of } x$$

If you are taking a square-root, you are raising to the one-half power, the relative uncertainty is one half of the number you are taking the square root of.

$$\textbf{relative uncertainty of } \sqrt{x} = \frac{\textbf{relative uncertainty of } x}{2}$$