## Rotational Equilibrium

As you work through the steps in the lab procedure, record your experimental values and the results on this worksheet. Use the exact values you record for your data to make later calculations.

## Rotational Equilibrium Lab - Simulation and Tools

Open the Rotational Equilibrium simulation to do this lab.

## Theory

## Torque

We define torque, $\tau$ (tau), as the product of the lever arm, $r_{\perp}$, and the force, $F$.

$$
\tau=r_{\perp} F
$$

Another quantity, which sadly has no name, is $r . r$ is the distance between the axis and the point of application of the force. Notice in Figures 2a-c that $r$ and $r_{\perp}$ are sometimes, but not always, equal.


Figure 2a: Torque on a rod.


Figure 2b: Force perpendicular to r. Maximum torque.


Figure 2c: Force collinear to r.
Zero torque.

1. Under what conditions are $r$ and $r_{\perp}$ equal but nonzero?

## Explore the Apparatus

9. Note the pulley attached to the side of the upward string. Don't touch it yet! What will happen to the beam if you push the pulley to the left? Why? Think about components of the tension in the string.
10. Will the result be different for a push to the left vs. a push to the right? (Assume equal angles.) Why?
11. At what angle relative to the beam would that force, $T_{y}$, be equal to half of $T$ ?
12. What buoyant force from the balloon will now be needed to balance the beam?

## I. Confirm that Torque $=$ Force Times Lever Arm for a Horizontal Lever

In Trials 1-3, you're given known positions for the right hanger. You'll then adjust its hanger weight to achieve balance.
3. In Trial 1, the given value of 1.00 m in the first position column tells you to add a second clamp and mass hanger (we'll call it "Hanger 2") at 100 cm . Then, following the order provided ( $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}$ ), determine each successive value theoretically, by calculation, and then experimentally.

Table 1: (Lever Arm)(Force)

| Rotational Equilibrium: $\mid$ Torque $_{\text {cw }}\|=\|$ Torque $_{\text {crw }} \mid$ |  |  |  | Weight, left end Lever arm, left end Torque, left end |  | $\begin{aligned} & 0.100 \mathrm{~g} \mathrm{~N} \\ & 0.500 \mathrm{~m} \\ & 0.0500 \mathrm{~g} \mathrm{~m} \mathrm{~N} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }_{\operatorname{ccw}}^{L}$ | it end, <br> ) torque |  | Rig | end, clockwi | (-) torque |  |
| Trial | Torque, $\tau$ $(g \mathrm{~m} \mathrm{~N})$ | Position (given) (m) | Lever arm (m) | $\begin{aligned} & \text { Weight } \\ & \text { (theoretical) } \\ & (g \mathrm{~N}) \end{aligned}$ | Weight <br> (exper.) $(g \mathrm{~N})$ | \% Difference (weights) |
| 1 | 0.0500 | 1.00 | (a) | (b) | (c) | (d) |
| 2 | 0.0500 | 0.60 | (a) | (b) | (c) | (d) |
| 3 | 0.0500 | 0.680 | (a) | (b) | (c) | (d) |

For Trial 3, show your calculations of the weight required to balance the beam.
$\begin{array}{l}\text { weight }_{\text {left }} \times \text { lever arm } \\ \text { weigt }\end{array}=$ weight (theoretical $)_{\text {right }} \times$ lever arm $(\text { given })_{\text {right }}$
4. In Trials 4-5, you're given known weights for the right hanger. You'll then adjust its hanger position to achieve balance.
5. In Trial 4, the known value of $0.200 g \mathrm{~N}$ for the weight tells you to adjust the weight on the right hanger to provide a total hanger weight of $0.200 g \mathrm{~N}$. Then, following the order provided ( $a, b, c, d$ ), determine each successive value.

Table 2: (Lever Arm)(Force)

| Rotational Equilibrium: $\mid$ Torque $_{\mathbf{c w}}\|=\|$ Torque $_{\text {ccw }} \mid$ |  |  |  | Weight, left end Lever arm, left end Torque, left end |  | $\begin{aligned} & 0.100 \mathrm{~g} \mathrm{~N} \\ & 0.500 \mathrm{~m} \\ & 0.0500 \mathrm{~g} \mathrm{~m} \mathrm{~N} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \mathrm{L} \\ \operatorname{ccw} \end{gathered}$ | end, <br> torque | Right end, clockwise (-) torque |  |  |  |  |
| Trial | Torque, $\tau$ ( $g \mathrm{~m} \mathrm{~N}$ ) | Position (from predicted lever arm) (m) | Lever arm (theoretical) (m) | Lever arm (exper.) (m) | Weight (given) ( $g \mathrm{~N}$ ) | \% Difference (lever arms) |
| 4 | 0.0500 | (b) | (a) | (c) | 0.200 | (d) |
| 5 | 0.0500 | (b) | (a) | (c) | 0.150 | (d) |

For Trial 5, show your calculations of the position of the hanger required to balance the beam.
weight $_{\text {left }} \times$ lever arm left $=$ weight $(\text { given })_{\text {right }} \times$ lever arm $(\text { theoretical })_{\text {right }}$

## II. Weighing an Unknown Object

2. The right-most mass in your mass set will act as our unknown. We want to determine its weight. Using the selector below the unknown mass, select any number from 2-10. (A particular number may be assigned to you.)

Selected unknown mass number:
7. Record your value for the unknown weight.
8. Show your calculations.

## III. Determine the Weight of the Beam

4. What total weight was required, including the clamp and hanger?
5. Calculate the weight of the beam.

Show your calculations of the weight of the beam.
6. Calculate the torque due to the weight of the beam.

Show your calculations of the torque due to the weight of the beam.
7. To find the weight of each end, we need the linear density, $\mu$. The linear density is the mass/length for the beam. You know both its mass and length now.

Show the calculations in this space.

$$
\mu=\operatorname{mass}(\mathrm{kg}) / \text { length }(\mathrm{m})=
$$

8. The left end of the beam is 0.200 m long. Using mass $=\mu \times$ length, determine the mass of the $20-\mathrm{cm}$ left end.
9. The center of gravity of the left end is at the center of this $20-\mathrm{cm}$ section. Record this value.
10. Similarly, find $m_{\text {right }}$ end and the lever arm of right end.
11. Calculate the ccw torque and cw torque with this new configuration of the beam. As before, we'll ignore the mass hangers. We just want the total torque due to the two ends of the beam.

Show your calculations of the positive counter-clockwise and negative clockwise torques and the net torque.
12. How do the results of the two methods of calculating the torque due to the weight of the beam compare?

## IV. The Torque Due to Forces Not Perpendicular to the Lever

8-15. Record your data and calculations in Table 3.
Table 3: Torque $=r \boldsymbol{F} \sin (\theta)=r \sin (\theta) F$

| Torque due to mass hanger at $30 \mathrm{~cm} \quad \mathrm{~cm} \mathrm{~m} \mathrm{~N}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Case I. $\tau=r \boldsymbol{F} \sin (\theta)=r F_{\perp}$ |  |  | Case II. $\tau=r \sin (\theta) F=r_{\perp} \mathbf{F}$ |  |
| Trial | $\begin{gathered} \Phi \\ \left(^{\circ}\right) \end{gathered}$ | $\begin{gathered} \theta \\ \left({ }^{\circ}\right) \end{gathered}$ | $\boldsymbol{F}_{\text {buoyant }}$ $(g \mathrm{~N})$ | $\begin{gathered} \boldsymbol{F}_{\perp}=\underset{(g \mathrm{~F})}{\boldsymbol{F}_{\mathbf{b}} \sin (\theta)} \\ (\boldsymbol{g}) \end{gathered}$ | $\begin{gathered} r \\ (\mathrm{~m}) \end{gathered}$ | $\begin{gathered} r F_{\perp} \\ (g \mathrm{~m} \mathrm{~N}) \end{gathered}$ | $\begin{gathered} r_{\perp}=r \sin (\theta) \\ (\mathrm{m}) \end{gathered}$ | $\begin{gathered} r_{\perp} F_{\mathrm{b}} \\ (g \mathrm{~m} \mathrm{~N}) \end{gathered}$ |
| 1 | 0 | 90 | 0.100 |  |  |  |  |  |
| 2 | 30 | 60 |  |  |  |  |  |  |
| 3 | 45 | 45 |  |  |  |  |  |  |
| 4 | 60 | 30 |  |  |  |  |  |  |

