

# Conservation of Momentum

## PURPOSE

- To investigate the behavior of objects colliding in elastic and inelastic collisions.
- To investigate momentum and energy conservation for a pair of colliding carts.
- To investigate the effect of the type of collision on the change in kinetic energy of a system of colliding carts.

## EQUIPMENT

Virtual Momentum Track

VPL Grapher

Pencil

## SIMULATION AND TOOLS

Open the Virtual Momentum Track<sup>1</sup> simulation to do this lab.

You will need to use the VPL Grapher<sup>2</sup> to complete this lab.

## EXPLORE THE APPARATUS

Open the Virtual Momentum Lab. A lot should look familiar after having worked with the Dynamics Track.

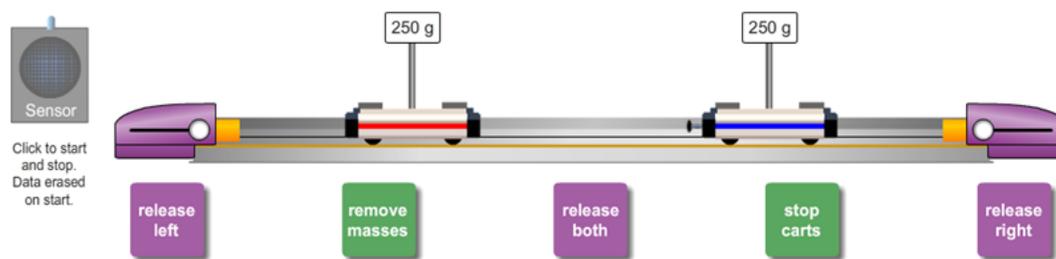


Figure 1: Carts, track, and motion sensor in the Virtual Momentum Lab simulation

Let's take a trial run with the apparatus.

Toss the carts around a bit to see how they interact. You'll see that they bounce nicely off of each other and lose a bit of speed when they collide with the orange bumpers. Turn on the "Velcro<sup>®</sup>" and try it again. Hopefully they act as you'd expect.

<sup>1</sup>../simulation/be4m4omb.swf

<sup>2</sup>../grapher.swf

Turn the “Spring Plunger” back on. Start both carts moving. Now click on either cart. The other one stops, too. But when you release the mouse button, the unclicked cart starts moving again. The best way to stop them both is with the **Stop Carts** button.

Try adding some mass to one cart. You have to release the mass while it’s over the mast (the rod holding up the flag). Bang the carts around a bit to see how the mismatched carts behave.

Now try the launchers. Here’s how they’re operated: Drag a cart to the launcher bumper and continue dragging it until it appears to go behind the bumper. Release the cart. This engages the cart and bumper. Now cock the launcher spring by pulling back the white circle on the side of the launcher. Click the **release** button to fire. Try again with the launcher spring pulled back different distances. Try launching both at once (**release both**).

This would be a good time to try out the “Motion Sensor.” It’s a bit trickier with this lab since you have two carts to work with, as well as abrupt changes in speed and direction. Try this: Double the mass of the red cart by adding 250 grams. Attach it to its launcher and pull back the spring about 50%. Put the blue cart at mid-track. Switch to the “Spring Plunger” if you still have the “Velcro<sup>®</sup>” on.

You’ll soon need to change the masses on the carts. You remove masses by double-clicking on the stack of masses. In computer lingo, the order is LIFO. “Last in, first out.” The last mass added (the top one) is the first one removed by clicking.

- Turn on the “Motion Sensor” by clicking it.
- Release the left launcher.
- Once the carts collide, turn off the “Motion Sensor.”
- You should see a graph similar to the one shown in Figure 2.

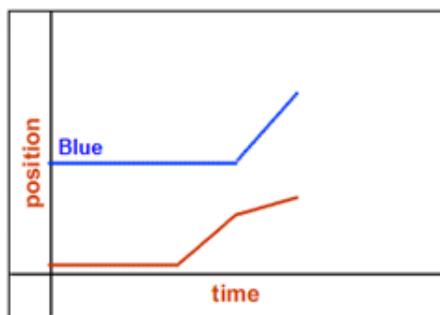


Figure 2

Let’s identify all the parts of this interaction on our graph.

When you turn on the “Motion Sensor,” each cart is stationary until you click the launcher **release** button. During this time each cart has an initial, constant position (zero velocity) as indicated by horizontal ( $t, x$ ) lines.

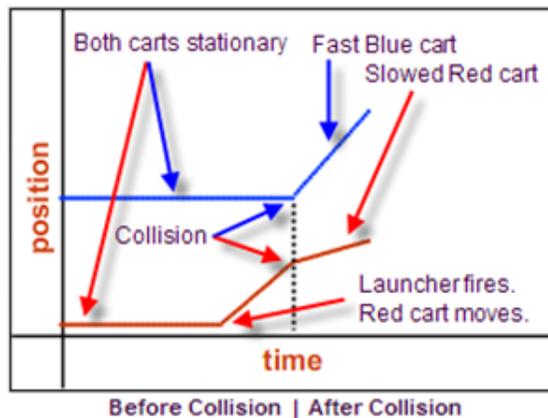


Figure 3

When the launcher is fired, the red cart is given an initial velocity. (We ignore acceleration in these graphs.)

The red cart collides with the blue cart.

The red cart is pushed to the left, slowing it down.

The blue cart is pushed to the right with an equal but opposite force, thus giving it a positive velocity which appears to be somewhat faster than the red cart's initial velocity.

Let's analyze our data. You have five velocities, but you're interested in only four of them. **You are only interested in the velocities just before and just after interactions—collisions.** The blue cart is at rest before, and traveling at some positive velocity after the collision. The red cart has a positive velocity before and a smaller positive velocity after the collision. The red cart's initial zero velocity before launch is not related to the collision. You'll always ignore similar segments.

The blue cart's initial zero velocity is obvious from the graph. Let's see how to find the other velocities using Grapher.

- 1 In the momentum apparatus, click **Copy Data to Clipboard**.
- 2 Open Grapher. Click in the "paste data" box and use Ctrl+V to paste it into the data table.
- 3 You should see three columns of data labeled "X," "Blue," and "Red." These hold the data for time, blue cart position, and red cart position, respectively. You should also see two graphs—one position-time graph for each cart. Use manual scaling to make sure that each graph starts at (0, 0) and has the same maximum  $x$  and  $t$  values. A range of 0.0 m to 2.0 m is good. Label the axes of the top graph "Blue Position (m)" and "Time (s)." Label the bottom graph similarly. Give each graph an appropriate name.
- 4 Turn on "Linear Fit." Determine the constant velocities of the carts prior to the collision by finding the four slopes of their  $(t, x)$  graphs. You should know how to use the "Linear Fit" tool from your kinematics labs or by studying the Lab Guide. You'll note that when you drag across a certain portion of the graph, a pink line of best fit is drawn for that range of points. The

data in the “Linear Fit” data box also corresponds to that data range. Thus you can determine before-and-after-collision velocities for each cart. The longer the line segment you drag across, the better your results will be.

You should now have something like Figure 4. (Actually, Figure 4 has been cobbled together from both graphs to make it easier to compare the graphs.) The actual velocity values depend on your initial plunger setting.

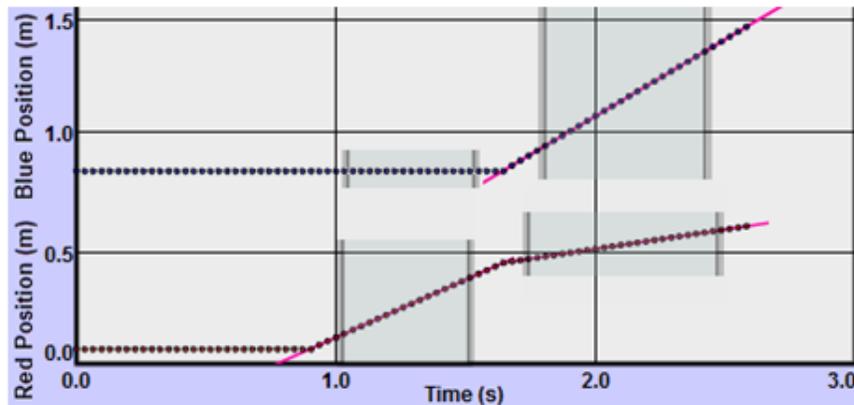


Figure 4

The slopes produced by Grapher are as follows.

$$\begin{aligned} \mathbf{V}_{o \text{ Blue}} &= 0 \text{ m/s} \\ \mathbf{V}_{o \text{ Red}} &= 0.517 \text{ m/s} \\ \mathbf{V}_{f \text{ Blue}} &= 0.686 \text{ m/s} \\ \mathbf{V}_{f \text{ Red}} &= 0.174 \text{ m/s} \end{aligned}$$

These velocities and the masses are the only values needed to analyze the momenta and energies of our carts.

## THEORY

### Momentum

In class and in your reading, you’ve seen the following development of the concepts of momentum, momentum conservation, and their connection to energy conservation. (Vectors are shown in bold type.)

The **momentum**,  $\mathbf{p}$ , of a body of mass,  $m$ , and velocity,  $\mathbf{v}$ , is written as

$$\mathbf{p} = m\mathbf{v}. \tag{1}$$

**Case 1: A net external force acts on a body.**

When a (net) force acts on a body, its velocity and hence its momentum will change according to equation 2.

$$\mathbf{F}\Delta t = \Delta\mathbf{p} \quad (2)$$

where  $\mathbf{F}\Delta t$  is referred to as the **impulse, J**.

Note that the impulse is a vector and is equal to the difference in two momentum vectors,  $\mathbf{p}_f$  and  $\mathbf{p}_o$ .

In words, equation 2 says

$$\text{impulse} = \text{change in the momentum.} \quad (3)$$

**Case 2: An internal force acts between two bodies (or more than two, but that's out of our league).**

As you know, forces come in equal and opposite pairs. So you know that in case 1 you were just focusing on one of the bodies involved. That's great for determining things like the effect of the force on a golf ball hit by a golf club.

There are also many situations where it's useful to look at both bodies involved. If you want to slow down or change the course of an incoming asteroid, you need to know how big an object you need to throw at it, how fast you need to throw it, and in what direction. In these cases, the impulse on the **system** is zero since all forces are internal to the system.

When there is a no net, external force,

$$\mathbf{F}\Delta t = \Delta\mathbf{p} = 0. \quad (4)$$

That is, there is no impulse, so there will be no change in momentum. Thus **the momentum of the system is conserved**.

$$\mathbf{P}_{(\text{sys})o} = \mathbf{P}_{(\text{sys})f} \quad (5)$$

**So, if there is no net, external force on the system, the momentum of the system will remain unchanged even if the parts of the system exert forces on one another and the individual parts change their momentum.**

An excellent example of this behavior is the system of two carts that you collided together earlier. Their total momentum before the collision,  $\mathbf{p}_o$ , was equal to their total momentum after the collision,  $\mathbf{p}_f$ . Try using the data in Figure 4 to verify the following.

For a **system of two carts**, you can expand  $\mathbf{p}_{(\text{sys})o} = \mathbf{p}_{(\text{sys})f}$  to

$$\mathbf{p}_{1o} + \mathbf{p}_{2o} = \mathbf{p}_{1f} + \mathbf{p}_{2f}. \quad (6)$$

After rearranging,

$$\Delta\mathbf{p}_1 = -\Delta\mathbf{p}_2. \quad (7)$$

That is, the loss in momentum by cart one equals the negative of the gain in momentum of cart two.

## Kinetic Energy

You've also become familiar with the **kinetic energy**, KE, of an object. The equation for kinetic energy involves the same terms,  $m$  and  $v$ .

$$\text{KE} = \frac{1}{2}mv^2 \quad (8)$$

So, shouldn't the kinetic energy of the system always be conserved, too? That is, shouldn't it be the same before and after an interaction? No. There's a subtle difference between the equation  $m\mathbf{v}$  for momentum and  $\frac{1}{2}mv^2$  for kinetic energy. The  $v$  in kinetic energy is just the speed, a scalar. So, kinetic energy is a positive scalar, and we could never add up two kinetic energies for a result of zero. Kinetic energy is not necessarily conserved. Momentum is always conserved.

In this lab, we'll observe the momentum of a system of two carts in several situations. We'll also investigate their kinetic energies.

## PROCEDURE

Please print the worksheet for this lab. You will need this sheet to record your data.

### I. Conservation of Momentum in a Collision

**NOTE:**

In most of this lab, you'll use your data to answer questions, even non-numerical questions.

Equations 6 and 7 say the same thing, but in two different ways. Let's explore both.

- 1 Load the blue cart with 500 g, giving it a total mass of 750 g. (Remove any mass that has been added to the red cart.)
- 2 Position the blue cart right between the **release both** and **stop carts** buttons.
- 3 Engage the red cart with its launcher. Cock the launch spring about halfway back.
- 4 Turn on the "Motion Sensor."
- 5 Release the left cart.
- 6 Somewhat before the blue cart hits the right bumper, turn off the "Motion Sensor." Your graph should look something like Figure 5.

There may be additional lines due to collisions with the plungers. Ignore them. Dragging the big white ball below the graph will adjust the time display scale.

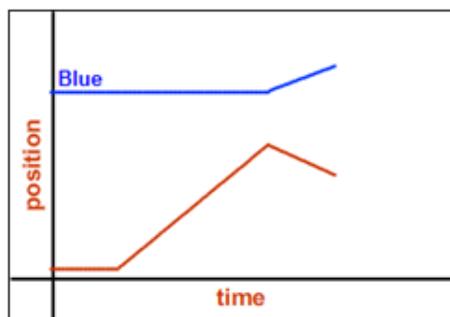


Figure 5

Before finding actual values for the velocities, let's think about what the graph tells us about the signs of the momenta.

- 7 What is the sign of the red cart's initial momentum,  $m_R \mathbf{v}_{R0}$ , its momentum just before the collision?
- 8 What is the sign of the red cart's final momentum,  $m_R \mathbf{v}_{Rf}$ , its momentum just after the collision?
- 9 What is the sign of the change in the red cart's momentum,  $\Delta(m_R \mathbf{v}_R) = m_R(\Delta \mathbf{v}_R)$ ?
- 10 The blue cart's initial momentum,  $m_B \mathbf{v}_{B0}$ , doesn't have a sign. What is its momentum before the collision?
- 11 What is the sign of the blue cart's final momentum,  $m_B \mathbf{v}_{Bf}$ , its momentum just after the collision?

12 What is the sign of the change in the blue cart's momentum,  $\Delta(m_B \mathbf{v}_B) = m_B(\Delta \mathbf{v}_B)$ ?

So the red cart had a negative change in momentum while the blue cart had a positive change in momentum. If the total momentum must be constant, then the sum of these changes, the total change, must add to zero. Let's look at the numbers.

13 Copy+Paste your data from the apparatus into Grapher as before. Using the "Linear Fit" tool, determine the carts' velocities before ( $\mathbf{V}_o$ ) and after ( $\mathbf{V}_f$ ) the collision. Record these in Table 1.

14 Using the known masses calculate and record the momentum of each cart before and after the collision (4 values).

15 Add the initial momenta of the carts to determine the initial momentum ( $\mathbf{p}_{(\text{sys})o}$ ) of the system. Similarly determine the final momentum ( $\mathbf{p}_{(\text{sys})f}$ ) of the system. Record these two values in Table 1.

16 Finally, calculate and record the change in momentum of the system ( $\Delta \mathbf{p}_{\text{sys}}$ ) from  $(\mathbf{p}_{(\text{sys})f}) - (\mathbf{p}_{(\text{sys})o})$ .

17 Equation 6 says that  $\mathbf{p}_{(\text{red})o} + \mathbf{p}_{(\text{blue})o} = \mathbf{p}_{(\text{red})f} + \mathbf{p}_{(\text{blue})f}$ . How did that work out? Quote your numbers.

18 Equation 7 says that  $\mathbf{p}_{(\text{red})f} - \mathbf{p}_{(\text{red})o} = -(\mathbf{p}_{(\text{blue})f} - \mathbf{p}_{(\text{blue})o})$ . How did that work out? Quote your numbers.

## II. Elastic Collisions

With the "Spring Plunger," the carts seemed to bounce nicely off of each other. Could this have been an elastic collision, that is, one where the kinetic energy of the system was unchanged during the collision? Let's do the math and see.

1 Table 2 works just like Table 1 and uses the same data. By filling in Table 2, you'll calculate the total KE of the system of two carts before ( $\text{KE}_{(\text{sys})o}$ ) and after ( $\text{KE}_{(\text{sys})f}$ ) the collision. You'll also calculate the change in kinetic energy of the system,  $\Delta \text{KE}_{\text{system}}$ , which is  $\text{KE}_{(\text{sys})f} - \text{KE}_{(\text{sys})o}$ . Note that in the equation for KE, only the speed is considered. That is, KE has no direction associated with it. This actually doesn't matter in your calculations since all the velocities are squared.

2 Does the *kinetic energy* of the system appear to be conserved in this collision? How do you know? Quote your numbers.

## III. Totally Inelastic Collisions

In the previous section, we observed carts making elastic collisions. We found that both momentum and kinetic energy were conserved. Our carts can have another type of collision. The springy bumpers between them can be replaced by sticky "Velcro<sup>®</sup>" bumpers. In this case we would have a **totally inelastic collision, one where the carts stick together after the collision and share a common speed.**

1 Let's start with the simplest case – a collision between carts of equal mass and equal but opposite velocities. Attach both carts to their launchers. Remove all extra masses. Fully cock both launchers (to provide equal initial speeds). Using the usual procedure, produce a graph of their head-on collision. You'll need to use the **release both** button.

a Copy and paste your data into Grapher. Add axis labels and an appropriate title to each graph. Manually adjust the  $y$ -axis scale on both graphs to range from 0.0 m to 2.0 m.

b Take a single **Screenshot**  of the graph from Grapher and upload it as "Mom\_III1.png".

c Draw the position-time graph produced by the apparatus in Graph III1.

d Explain how you know from the graph(s) that **momentum was conserved, but kinetic energy was not**. You should be able to tell just by looking at the graphs, but you can use Grapher to assist you.

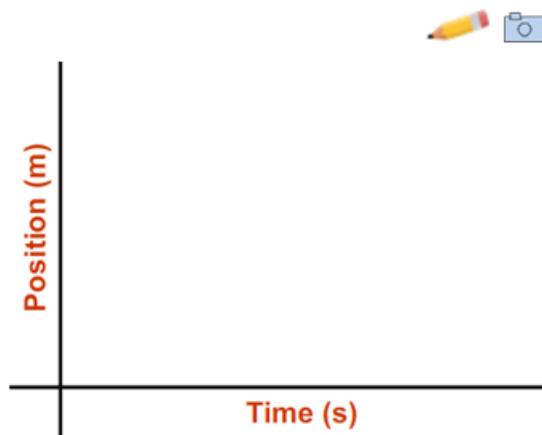


Figure Graph III1: Head on Equal speed collision

2 Now try a similar collision, but this time with the blue cart initially sitting still at the center of the track.

a Copy and paste your data into Grapher. Again manually adjust the  $y$ -axis scale on both graphs to range from 0.0 m to 2.0 m. Add axis labels and an appropriate title to each graph.

b Take a single **Screenshot** of the pair of graphs from Grapher and upload it as "Mom\_III2.png".

c Draw the position-time graph produced by the apparatus in Graph III2.

d This one's not so obvious. You'll need some data and calculations to work with. Use Grapher to get the data you need. Use Table 3 to record your data and calculated values.

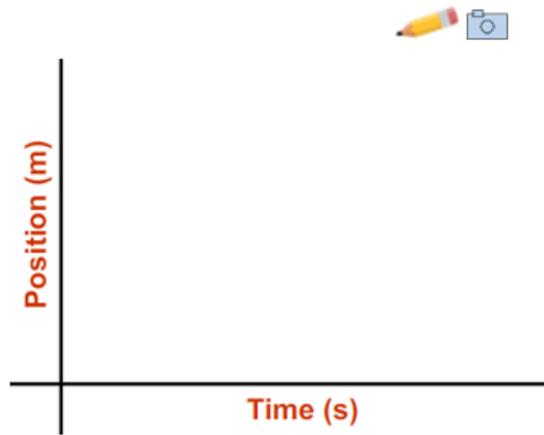


Figure Graph III2: Collision with stationary cart

- 3 Using your data, explain how you know that momentum was conserved, but kinetic energy was not conserved.
- 4 You should have found that the pair of carts moving as one with a mass of  $2 \times m_{\text{cart}}$  had half the speed of the single cart with a mass of  $1 \times m_{\text{cart}}$ . Why then is the kinetic energy less after the collision? **That is, why doesn't one cart with a speed of  $v$  have the same kinetic energy as two carts each with a speed of  $\frac{v}{2}$ ?** (Here's a head start to your answer: Before the collision, the object carrying all the momentum and KE had a mass of  $m_{\text{cart}}$  and a speed of  $V_o$ .)
- 5 For the situation in Graph III1, what fraction of the kinetic energy of the carts remains after the collision? That is, what is  $(KE_{\text{final}}/KE_{\text{initial}})$ ? You don't have any numbers to work with, but you don't need any.
- 6 For the situation in Graph III2, what fraction of the kinetic energy of the carts remains after the collision? That is, compute  $(KE_{\text{final}}/KE_{\text{initial}})$ . You'll need to use your numbers this time.