

The Force Table – Vector Addition and Resolution

“Vectors? I don’t have any vectors. I’m just a kid.” – From *Flight of the Navigator*

PURPOSE

- To observe how forces acting on an object add to produces a net, resultant force.
- To see the relationship between the equilibrant force relates to the resultant force.
- To develop skills with graphical addition of vectors and vector components.
- To develop skills with analytical addition of vectors and vector components.

EQUIPMENT

Force Table simulation

Pencil

SIMULATION AND TOOLS

Open the Force Table¹ simulation to do this lab.

See the Video Overview².

EXPLORE THE FORCE TABLE APPARATUS; THEORY

We’ll use the force table apparatus in this lab activity.

NOTE:

For each direction answer, enter an angle between 0° and 360° with respect to the force table.

NOTE:

Working with vectors graphically is somewhat imprecise. You should expect to see lower precision in this lab. But working with vectors graphically will give you a better understanding

¹../f2c4tb8lv2b.swf

²<http://virtuallabs.ket.org/physics/apparatus/04.1stequil/>

of vector addition and vector components. You can improve your precision by using the Zoom features of the apparatus.

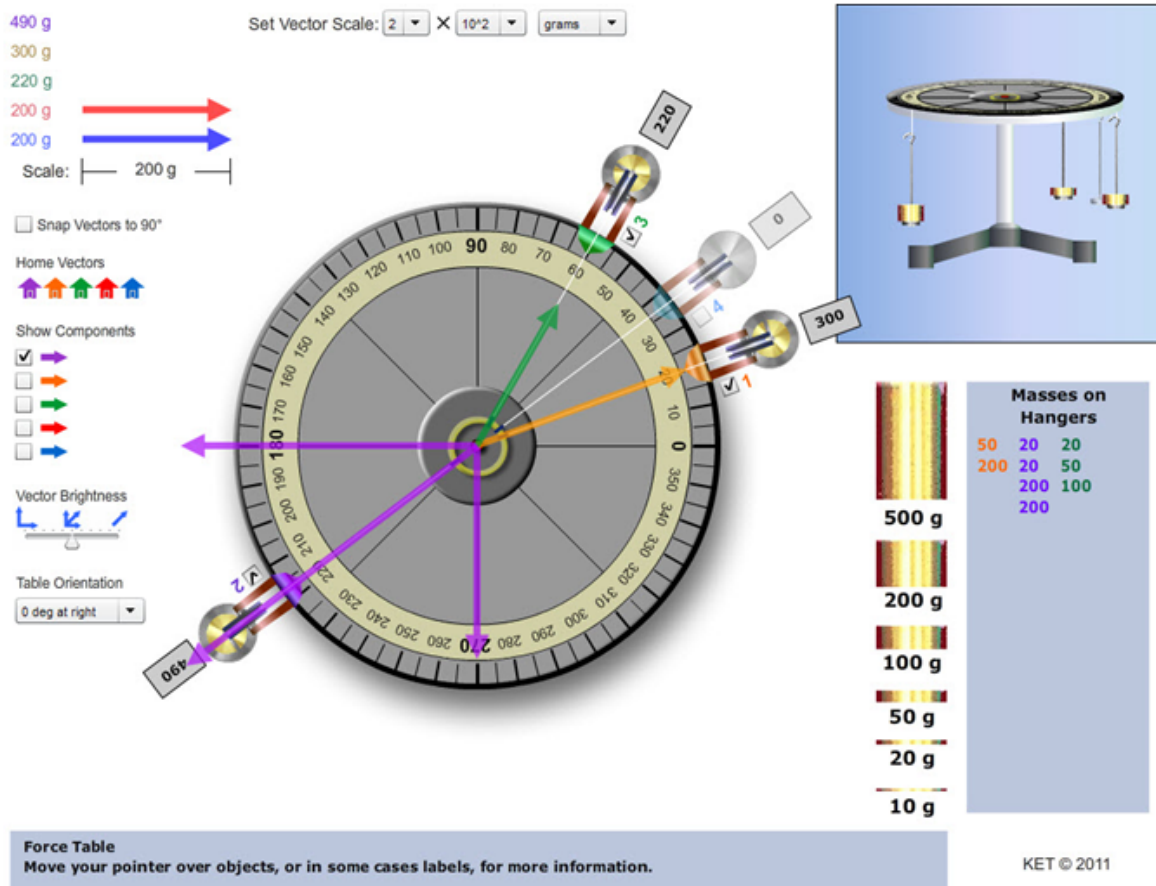


Figure 1: The Force Table Apparatus

NOTE:

When you're asked to provide a figure, you may sketch it and scan a copy or take a picture of it; or you may create it with the force table apparatus, do a print screen, and save it as a file. Whatever file you create, you will need to upload a copy in WebAssign.

PROCEDURE

Please print the worksheet for this lab. You will need this sheet to record your data.

I. Addition of Two-Dimensional Vectors

In this part, you'll add two forces to find their resultant using *experimental addition*, *graphical addition*, and *analytical addition*. In WebAssign, you will randomly be given two forces to work with.

There are two things that we should clarify about these two statements. We'll refer to Figure 1 in the following.

- The force subscripts on forces and angles (\mathbf{F}_2 , θ_2) refer to the hanger numbers. For example, in Figure 1, $\theta_1 = 20^\circ$.
- The notation for forces in this lab is a bit unusual. From the "Masses on Hangers" table, we know that hanger #1 has a 50 g and a 200 g mass on it. The hanger's mass is 50 g, so the total, 300 g, is shown in the box beside the hanger.

We want to record forces in newtons. From $W = mg$, we can easily calculate the weight of a 300 g mass.

$$W = 0.300 \text{ kg} \times 9.8 \text{ N/kg} = 2.94 \text{ N}$$

But there's a much more convenient way of handling this. We can abbreviate this calculation to the simpler $W = 0.300 \text{ gN}$.

A. Experimental Addition; The Equilibrant

By *experimental addition* we mean that we will actually apply the two forces to an object and measure their total, resultant, effect. We'll do that by first finding the *equilibrant*, \mathbf{E} , the force that exactly balances them. When this equilibrium is achieved, the ring will be centered. The *resultant*, \mathbf{R} , is the single force that exactly balances the equilibrant. That is, either \mathbf{F}_2 and \mathbf{F}_4 , or their resultant \mathbf{R} can be used to balance the equilibrant.

NOTE:

When adjusting the angle of a pulley to a prescribed angle, pay no attention to the string. Drag the pulley until the color-coded pointer is at the desired setting. Another way that works well is to disable all the hangers while adjusting the angles. When you do this, the ring is automatically centered so the strings can be used to help adjust the angle.

- 1 Enable pulleys 2 and 4 and disable pulleys 1 and 3 using their check boxes. A disabled pulley is grayed out. You can drag it anywhere you like since any masses on it are inactive.
- 2 Drag pulleys 2 and 4 to their assigned angles, θ_2 and θ_4 . Use the purple and blue pointers of the pulley systems, not the strings, to set the angles. You should zoom in to set the angles more precisely.
- 3 Add masses to hangers 2 and 4 to produce forces \mathbf{F}_2 and \mathbf{F}_4 .
- 4 Enable pulley 3.
- 5 Using pulley 3, experimentally determine the equilibrant, \mathbf{E} to just balance (cancel) \mathbf{F}_2 and \mathbf{F}_4 . Do this by adjusting the mass on the hanger and the angle until the yellow ring is approximately centered on the central pin. You can add and remove masses to home in on it. You add masses by dragging and dropping them on a hanger. You remove them using the **Mass Total/Mass Removal** tool. This tool displays the total mass on the hanger, including the hanger's mass. Clicking on the tool removes the top mass on the hanger.



Figure 2: MT/MR Tool

- 6 From your value of \mathbf{E} , what should be the resultant of \mathbf{F}_2 and \mathbf{F}_4 ? (Same force, opposite direction.)
- 7 Disable pulleys 2 and 4 and enable pulley 1. Experimentally determine the resultant by adjusting the mass on pulley 1 and its angle until it balances the equilibrant. That is, the ring should not move substantially when you change between enabling \mathbf{F}_2 and \mathbf{F}_4 , and enabling just \mathbf{F}_1 .

B. Graphical Addition

- 1 With *graphical addition*, you will create a scaled vector arrow to represent each force. We add vector arrows by connecting them together tail to head in any order. Their sum is found by drawing a vector from the tail of the first to the head of the last. Using the default 2×10^2 g vector scale, create vector arrows (± 3 grams) for \mathbf{F}_2 and \mathbf{F}_4 . For example, drag the purple vector by its body and drop it when its tail (the square end) is near the central pin. It should snap in place.

You now want to adjust its direction and length to correspond to direction and magnitude of \mathbf{F}_2 . It's hard to do both of these at the same time. That's where the "Resize Only" and "Rotate Only" tools come into play. They let you first adjust the direction and then adjust the length. Let's do \mathbf{F}_2 . Zoom in. Drag the head (tip) of the vector arrow in the assigned direction force \mathbf{F}_2 . That is, θ_2 . Zoom out. Click the "Resize Only" check box. Its direction is now fixed until you uncheck that box. You can now drag the head of the arrow and adjust just its magnitude without changing its direction. The current magnitude of \mathbf{F}_2 is displayed at the top of the screen next to the home location of that purple vector. Repeat for \mathbf{F}_4 using the blue arrow.

- 2 The resultant, \mathbf{R} , is the vector sum of \mathbf{F}_2 and \mathbf{F}_4 . Add \mathbf{F}_2 and \mathbf{F}_4 graphically by dragging the body of \mathbf{F}_4 until its tail is over the tip of \mathbf{F}_2 and releasing it. Form the resultant, \mathbf{R} , by creating an orange vector, \mathbf{F}_1 , from the tail of the first (vector \mathbf{F}_2) to the head of the last (vector \mathbf{F}_4). (You can actually connect \mathbf{F}_2 and \mathbf{F}_4 in either order.) You can read the magnitude and direction of \mathbf{R} directly from the length and direction of \mathbf{F}_1 . You'll likely find some disagreement between these results and your results from Part A. This is because both methods are somewhat imprecise.
- 3 The equilibrant, \mathbf{E} , should be the same magnitude as \mathbf{R} , but in the opposite direction. Produce \mathbf{E} with the green vector arrow. It should be attached to the central pin and in the direction of the force \mathbf{F}_3 .
- 4 Draw each of the four vector arrows on Figure 2.

Label each vector with its total force value. (Ex. Label the blue \mathbf{F}_4 vector 0.180 gN .)

Vectors \mathbf{F}_2 and \mathbf{F}_4 should be shown added in their tail-to-head arrangement. The \mathbf{E} and \mathbf{R} vectors should be shown radiating out from the central pin. Label each vector with its force value in gN units.

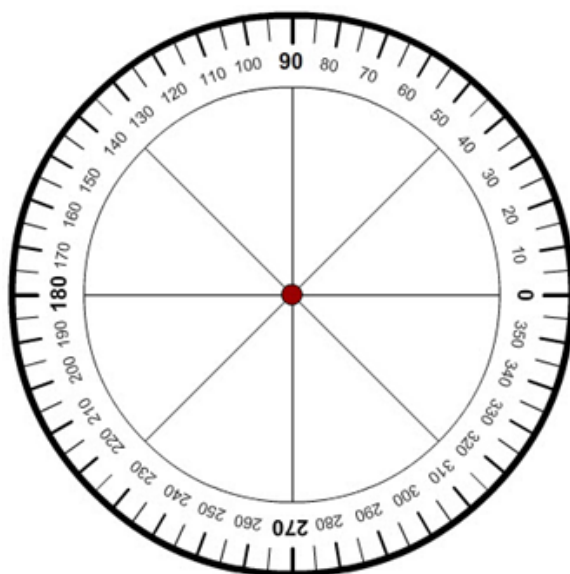


Figure 3: Graphical Addition; Equilibrant

FILE UPLOADS OF GRAPHICAL REPRESENTATIONS:

For the graphical representations in this lab, you have several choices for creating the files to upload. A couple of these are listed below. Regardless of the method you use, ensure that your file is in a format that your instructor can open.

- 1 Draw the vectors by hand.
 - a Use Figure 3 if it is best drawn on the force table schematic.

- b** Either scan your image in and save it as a file or take a picture of your drawing.
- 2** Do a print screen of your drawing from the simulation and save it as a file.

- 5** Describe what we mean by the terms resultant and equilibrant in relation to the forces acting in this experiment.
- 6** From your figures, which two forces when added would equal zero?
- 7** Show this graphical addition. You'll want to offset them a bit since they would otherwise be on top of one another. You will upload a file showing your graphical representation of this addition.
- 8** What three vectors when added would equal zero?
- 9** Show this graphical addition. You will upload this as a file.
- 10** Vector **E** should appear in both of these last two vector additions. Why?
- 11** Put all four vectors back at the center of the force table with their tails snapped to the central pin. Click the check boxes beside the purple and blue arrows under "Show Components." This will give you a visual check on your analytical calculations of the component values in Part II.A3. So be sure to look back at the figure as you do Part II.A3.

C. Analytical Addition with Components

You've found experimentally and graphically how to add the forces \mathbf{F}_2 and \mathbf{F}_4 to produce the resultant, \mathbf{R} . Hopefully you found that these two methods make it very clear to you what is meant by the addition of vectors. But you also found that both methods are pretty imprecise—sort of the stone tools version of vector addition. We can get much better results using trigonometry.

You've just verified that the resultant effect of two (or more) vectors can be found by attaching them together tail to head and drawing a new vector from the tail of the first to the head of the last. That single new vector is equivalent to the two original vectors acting together. So the one achieves the same result as the original two. The reverse is also true. We can simplify the process of addition of vectors if we replace each vector with a pair of vectors whose sum equals the original vector. At least if we do it strategically.

- 1** Put all four vectors back at the center of the force table with their tails snapped to the central pin. Click the check box beside the purple arrow under "Show Components." Zoom in for a better look. The two new purple vectors are the x and y components of the purple vector, \mathbf{F}_2 . We would call them \mathbf{F}_{2x} and \mathbf{F}_{2y} .
- 2** Drag the x -component, \mathbf{F}_{2x} and add it to the \mathbf{F}_{2y} . That is, connect its tail to the head of \mathbf{F}_{2y} . Clearly they add up to \mathbf{F}_2 . But just to be sure, drag \mathbf{F}_{2x} back to the center and add \mathbf{F}_{2y} to it. Same result.

$$\text{So } \mathbf{F}_2 = \mathbf{F}_{2x} + \mathbf{F}_{2y}.$$

Note that this is all vector addition. We're using bold text for our vector names to emphasize that this is not scalar addition, which doesn't take direction into account. \mathbf{F}_2 equals the vector sum of \mathbf{F}_{2x} and \mathbf{F}_{2y} because when we connect the components together tail to head, the vector from the tail of the first to the head of the last is \mathbf{F}_2 .

So, we can use \mathbf{F}_2 and $\mathbf{F}_{2x} + \mathbf{F}_{2y}$ interchangeably. That's the strategic part.

- 3 Turn on the blue components.
- 4 We'll dim the main vectors. Drag the large blue dragger on the "Vector Brightness" tool almost all the way to the left. Only the components will remain bright. Leaving one of them, say, \mathbf{F}_{2x} where it is, add the other three components to it in any order you like. This might be a good time to **Hide Table**. There's a button at the top. Feel free to hide and show the force table as needed.
- 5 Notice where the head of the last component ends up. Notice how the four components add up to \mathbf{R} ! Try another order of addition. The order doesn't matter. So we can say,

$$\mathbf{R} = \mathbf{F}_1 = \mathbf{F}_{2x} + \mathbf{F}_{2y} + \mathbf{F}_{4x} + \mathbf{F}_{4y}$$

in any order.

Remember, this is vector addition, not scalar addition. So we're still having to connect them together graphically to find the resultant. We're still using stone tools.

- 6 Turn on the components for \mathbf{F}_1 . You now have three sets of components.
- 7 Drag all the y -components out of the way.
- 8 Add \mathbf{F}_{2x} and \mathbf{F}_{4x} together.
- 9 How do these two vectors relate to \mathbf{F}_{1x} ; that is, \mathbf{R}_x ?
- 10 Repeat with the y -components. What similar statement can you make about the y -components?
- 11 Finally, how is \mathbf{R} related to \mathbf{R}_x and \mathbf{R}_y ?

To summarize,

$$\mathbf{R}_x = \mathbf{F}_{2x} + \mathbf{F}_{4x} \text{ and } \mathbf{R}_y = \mathbf{F}_{2y} + \mathbf{F}_{4y} \text{ and } \mathbf{R} = \mathbf{R}_x + \mathbf{R}_y.$$

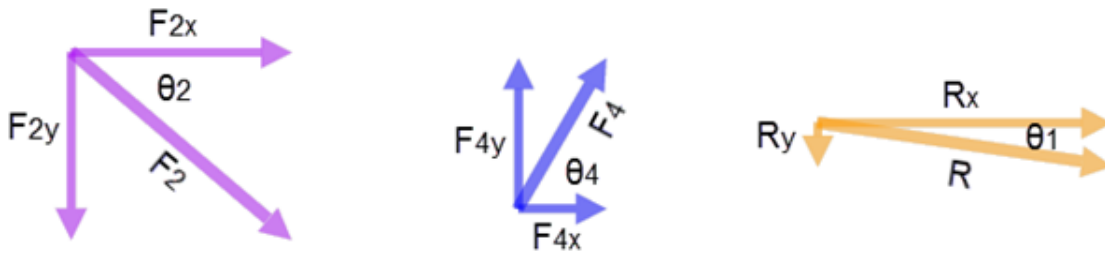


Figure 4: Vector Components

We can now leave our stone tools behind and take advantage of this new formulation by using trigonometric functions and the Pythagorean Theorem.

Here's our task. You were previously asked to find the resultant, \mathbf{R} , of \mathbf{F}_2 and \mathbf{F}_4 graphically. You now want to find the same result without the using the imprecise graphical methods.

Knowing \mathbf{F}_2 and θ_2 , we can calculate \mathbf{F}_{2x} and \mathbf{F}_{2y} analytically as follows.

$$\mathbf{F}_{2x} = \mathbf{F}_2 \cos(\theta_2) \quad (1)$$

$$\mathbf{F}_{2y} = \mathbf{F}_2 \sin(\theta_2) \quad (2)$$

We can do the same for \mathbf{F}_4 . We can then find \mathbf{R}_x and \mathbf{R}_y using

$$\mathbf{R}_x = \mathbf{F}_{2x} + \mathbf{F}_{4x} \quad (3)$$

$$\mathbf{R}_y = \mathbf{F}_{2y} + \mathbf{F}_{4y} \quad (4)$$

$$\mathbf{R} = \mathbf{R}_x + \mathbf{R}_y \quad (5)$$

There's one slight problem with Equation 5. Like Equations 3 and 4, it's a vector equation, but in Equations 3 and 4, the vectors are collinear, so they can be added with signs indicating direction. But since vectors \mathbf{R}_x and \mathbf{R}_y are perpendicular, we have to use the Pythagorean Theorem instead. So we can find the magnitude and direction of the resultant, \mathbf{R} , with the magnitudes of \mathbf{R} 's components.

$$\mathbf{R}^2 = \mathbf{R}_x^2 + \mathbf{R}_y^2 \quad (6)$$

$$\tan(\theta) = \frac{\mathbf{R}_y}{\mathbf{R}_x} \quad (7)$$

- 12 You've found experimentally and graphically how to add the forces \mathbf{F}_2 and \mathbf{F}_4 to produce the resultant, \mathbf{R} . Now let's try analytical addition. Using the table provided, find the components of \mathbf{F}_2 and \mathbf{F}_4 and add them to find the components of \mathbf{R} . Use these components of \mathbf{R} to determine the magnitude and direction of \mathbf{R} . Note that one of the four components will have a negative sign. The components now displayed on the force table should make it clear why.
- 13 Show **all** your calculations leading to your value for \mathbf{R} . Remember, a vector has both a magnitude and a direction. A summary of the steps is provided to get you started.

EXAMPLE:

$$F_{2x} = F_2 \cos(\theta_2) = 0.300gN \cos(\theta_2) \qquad F_{2y} = F_2 \sin(\theta_2) = 0.300 gN \sin(\theta_2)$$

$$R_x = F_{2x} + F_{4x} \qquad F_y = F_{2y} + F_{4y} \qquad R^2 = R_x^2 + R_y^2 \qquad \tan(\theta) = \frac{R_y}{R_x}$$

II. Simulation of a Slackwire Problem

Let's model a realistic system similar to what you might find in your homework. Figure 5 shows a crude figure of a slackwire walker, Elvira, making her way across the wire rope. At a certain instant, the two sides of the rope are at the angles shown. Only friction allows her to stay in place. The gravitational force is acting to pull her down the steeper "hill." If she were on a unicycle, she'd tend to roll to the center. It's complicated.

When friction is holding her in place, the single rope acts like two separate sections of rope in this situation with different tension forces on either side of her. To understand this, it helps to imagine her at the extreme left where the left section of rope is almost vertical and the right one is much less steep. In this case, \mathbf{T}_3 is providing almost all of the vertical support, while \mathbf{T}_1 pulls her a little to the right. It helps to just try it. Attach a string between two objects in the room. Leave a little slack in it. Now pull down at various points. You'll feel the big frictional tug on the more vertical side and less from the more horizontal side.

We want to explore this by letting her move along the rope.

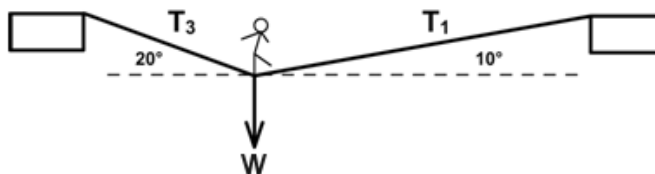


Figure 5: Elvira Off-Center on the Slackwire

A. Experimental Determination of \mathbf{T}_1 and \mathbf{T}_3

- 1 Preliminary prediction: Which tension do you think is greater, \mathbf{T}_1 or \mathbf{T}_3 ?
- 2 First send each of your vectors “home” by clicking on each of their little houses. Then remove all the masses from your four pulleys by activating all of them using their click boxes and then clicking in the total mass boxes beside each hanger until they read 50.
- 3 You will be given a randomized value for Elvira’s weight in your WebAssign question; use that value in the following. We’ll let 1 gram represent 1 newton on our force table and set our vector scale to 4×10^2 N. (Note the label to the left of each vector.) We’ll picture our force table as if it were in a vertical plane with 270° downward and 90° upward. We’ll use pulleys 1, 3, and 2 to provide our two tensions, \mathbf{T}_1 and \mathbf{T}_3 , and the weight of Elvira, respectively.
Disable all pulleys and set all the angles to match Figure 5. Use Zoom.
Turn pulleys 1, 3, and 2 back on.
- 4 Prediction: Since $\theta_3 = 2 \times \theta_1$, do you think \mathbf{T}_3 will be about twice \mathbf{T}_1 ?
- 5 For Elvira, move hanger 2 to 270° and set its total mass to x g to represent x N, where x N is Elvira’s weight.
- 6 Adjust the masses on each pulley until you achieve equilibrium. It’s best to alternate adding one mass to each side in turn until you get close to equilibrium.
- 7 Record the tensions.
- 8 How’d that prediction for the relationship between \mathbf{T}_1 and \mathbf{T}_3 go? If trig functions were linear, we wouldn’t need them.
- 9 Write a statement about the relationship between the steepness of the rope and the tension in that rope for this vertical arrangement.

B. Graphical Check of Your Results

- 1 Using the vector scale of 4×10^2 N, create vector arrows for each force, \mathbf{T}_3 , \mathbf{T}_1 , and \mathbf{W} (Elvira’s weight).
- 2 Upload a file of your graphical representation of these three forces.

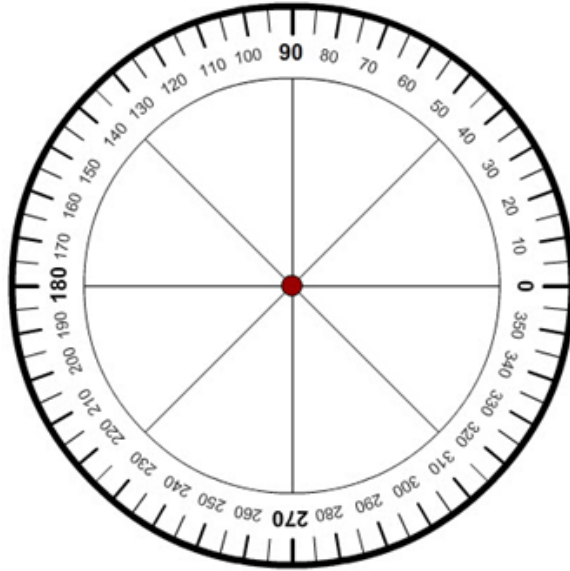


Figure 6: Elvira – Asymmetrical Vectors

- 3 How can we graphically check to see if our values for \mathbf{T}_1 and \mathbf{T}_3 are reasonably correct? How are \mathbf{T}_1 , \mathbf{T}_3 , and \mathbf{W} related? What would happen if any one of them suddenly went away? Elvira would no longer be in what?
- 4 Thus, the three vectors are in equilibrium and must add to equal zero! What would that look like? The sum, resultant, of three vectors is a vector from the tail of the first to the head of the last. If they add up to zero, then the resultant's magnitude would be zero, which means that the tip of the final vector would lie at the tail of the first vector.
- 5 Try it in the apparatus. Leave \mathbf{T}_1 where it is and then drag \mathbf{T}_3 's tail to \mathbf{T}_1 's head. Then drag \mathbf{W} 's tail to the head of \mathbf{T}_3 .
- 6 What about your new figure says (approximately) that the three forces are in equilibrium?
- 7 Draw your new figure with the three vectors added together.
- 8 Upload a file of your graphical representation of this vector addition. Don't forget to use the "Resize Only" tool after you get the angles set.

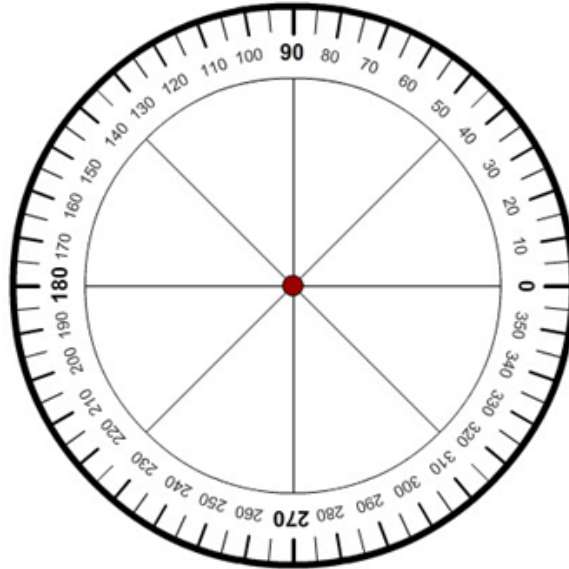


Figure 7: Graphical Addition – Asymmetrical

C. Analytical Check of Your Results

If our three vectors add up to zero, then what about their components?

- 1 When you add up the x -components of all three vectors, the sum should be what?
- 2 When you add up the y -components of all three vectors the sum should be what?
- 3 Test your predictions by calculating all the components and adding them. Don't forget the direction signs!

Before we continue . . . Why all the tension?

Why does the tension on each side have to be so much larger than the weight actually being supported? All the extra tension is being supplied by the x -components. Then why not just get rid of it? You'd have to move the supports right next to each other to make both ropes vertical. That would be pretty boring, and, frankly, nobody would pay to see such an act, even if they tried calling it "Xtreme Urban Slackwire."

Similarly, with traffic lights, a huge amount of tension is required to support a fairly light traffic light. The simpler solution of a pair of poles in the middle of the intersection wouldn't go over much better than Xtreme Urban Slackwire.

Next time you see large poles or towers supporting large electrical wires, look for places where the wire has to change direction (south to east, for example). The support structures in straight stretches (in-line) don't have to be really sturdy since they have horizontal forces pulling equally in opposite directions. Thus, they just have to support the weight of the wire. When the wires have to change directions, the poles at the corners have to provide these horizontal forces. Thus, they're much sturdier and larger to give them a wider base. They often have guy wires to the ground to help provide these forces. To make things more difficult, the guy wires will be usually be very steep, so they have to have a lot of extra tension in them to supply the horizontal force components.

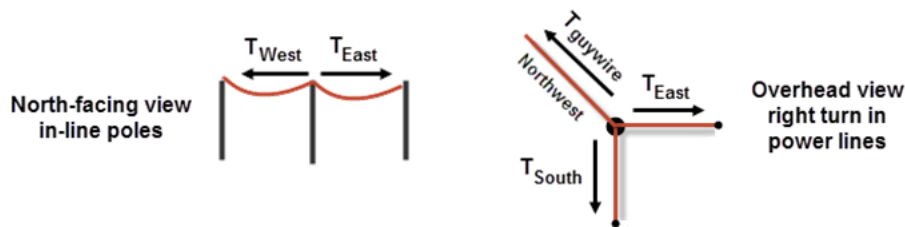


Figure 8: Power Lines – In Line and at Corners

III. Simulation of a Symmetrical Slackwire Problem

In Figure 9, Elvira has reached the center of the wire. This is where she would be if she rode a unicycle and just let it take her to the “bottom.” The angle values are just guesses. They’d be between the 10° and 20° we had before, but the actual value would depend on the length of the wire and its elastic properties.

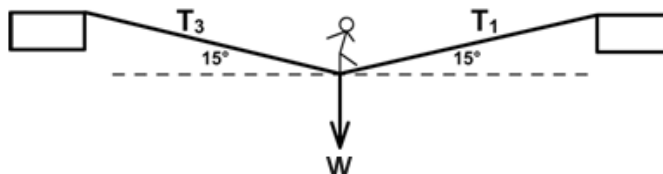


Figure 9: Elvira at the Center of the Slackwire

- 1 Without changing the masses used in Part II, adjust both angles to 15° .
- 2 What does the *symmetry* of the figure suggest about how the tensions, \mathbf{T}_1 and \mathbf{T}_3 , should compare?
- 3 Similarly, what can you say about *comparative* values of the *x*-components, T_{1x} and T_{3x} ? (Ex. Maybe $T_{1x} = 2 \times T_{3x}$?)
- 4 What can you say about *comparative* values of the *y*-components, T_{1y} and T_{3y} ?
- 5 Given that the weight being supported is W , what can you say about *actual* values of the *y*-components, T_{1y} and T_{3y} ?
- 6 You can now easily calculate \mathbf{T}_1 and, hence, \mathbf{T}_3 .
- 7 Change each \mathbf{T} to as close as you can get to this amount. Does this produce equilibrium?
- 8 With your apparatus, create the two vector arrows to match this amount—one for \mathbf{T}_1 and one for \mathbf{T}_3 . Add $\mathbf{T}_1 + \mathbf{T}_3 + \mathbf{W}$ graphically.
- 9 Upload a file with your graphical representation of this vector addition.

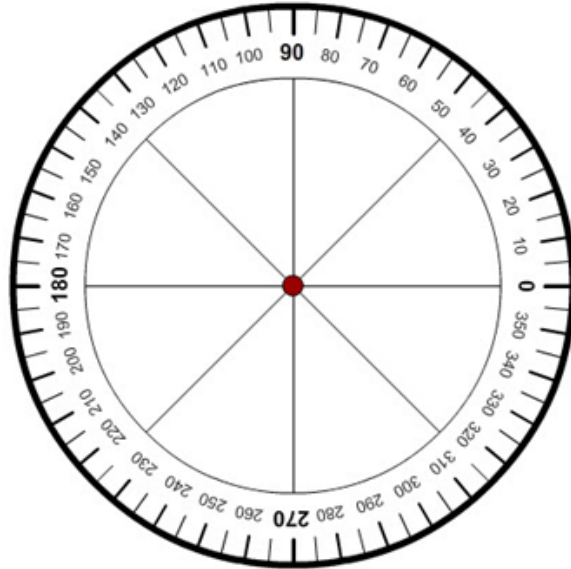


Figure 10: Graphical Addition – Symmetrical

You'll notice that this figure doesn't differ much from the previous one. The graphical tool we're using is not very precise. The same goes for "the real world." Measuring the tension in a heavy electrical cable or bridge support cable is very difficult. One good method involves whacking it with a hammer and listening for the note it plays!

IV. Vector Subtraction

Here's the scenario. We have a three-person kinder, gentler tug-o-war. The goal is to reach consensus, stalemate. Two of our contestants are already at work.

$$\mathbf{Darryl}_1 = 800 \text{ N at } 0^\circ$$

$$\mathbf{Darryl}_2 = 650 \text{ N at } 240^\circ$$

The question is—how hard, and in what direction, must **Larry**₃ pull to achieve equilibrium?

- 1 As before, empty all the hangers, and then set up pulleys 1 and 2 to represent these forces using 1 gram to represent 1 newton.
- 2 Send all the vector arrows home to get a clean slate. Then create orange and purple vectors to match, using 4×10^2 N for your vector scale. Attach them to the central pin.

One method of solving the problem is to add the Darryl forces and complete the triangle to find the resultant. Larry's force would be the equilibrant in the other direction. Another way is to add the two Darryl forces and then draw a third force, Larry, to complete the triangle which would leave a resultant of zero.

The following vector equation describes that method.

$$\Sigma \mathbf{F} = \mathbf{Darryl}_1 + \mathbf{Darryl}_2 + \mathbf{Larry}_3 = 0 \quad (8)$$

This is a vector equation. It means that you'll get a resultant of zero if you connect the three vectors together tail to head. To find the Larry vector, we need to subtract the two Darryl vectors from both sides. We know how to add vectors but how do we subtract them?

With **scalar math**, it would look like this:

$$\begin{aligned} \Sigma \text{Worth} &= \text{Darryl}_1\$ + \text{Darryl}_2\$ + \text{Larry}_3\$ = 0 && \text{(Yes, that's net worth.)} \\ \text{Larry}_3\$ &= -\text{Darryl}_1\$ - \text{Darryl}_2\$ \end{aligned}$$

If $\text{Darryl}_1\$ = \800 and $\text{Darryl}_2\$ = \650 , then we'd get

$$\begin{aligned} \text{Larry}_3\$ &= -\$800 - \$650 && \text{(Note that we're adding negatives here.)} \\ \text{Larry}_3\$ &= -\$1450. \end{aligned}$$

Negative dollars don't exist, but we would interpret this as something like a debt. That is, to make the three brothers have zero net worth, Larry_3 needs to be \$1450 in debt.

With vectors it works the same way, but we interpret the negative signs as indications of direction. A pull of -50 N to the left means a pull of 50 N to the right.

$$\begin{aligned} \Sigma \mathbf{F} &= \mathbf{Darryl}_1 + \mathbf{Darryl}_2 + \mathbf{Larry}_3 = 0 \\ \mathbf{Larry}_3 &= -\mathbf{Darryl}_1 - \mathbf{Darryl}_2 \\ \mathbf{Larry}_3 &= (-\mathbf{Darryl}_1) + (-\mathbf{Darryl}_2) \end{aligned}$$

So to find \mathbf{Larry}_3 , we need to create the two negative Darryl vectors and add them.

This means to draw the vectors $(-\mathbf{Darryl}_1)$ and $(-\mathbf{Darryl}_2)$, which are the opposites of \mathbf{Darryl}_1 and \mathbf{Darryl}_2 , and add them. That is, we can subtract vectors by adding their negatives.

So, if

$$\mathbf{Darryl}_1 = 800 \text{ N at } 0^\circ$$

$$\mathbf{Darryl}_2 = 650 \text{ N at } 240^\circ,$$

then the negatives of these are vectors of the same lengths but in the opposite directions. Thus,

$$-\mathbf{Darryl}_1 = 800 \text{ N at } 180^\circ$$

$$-\mathbf{Darryl}_2 = 650 \text{ N at } 60^\circ.$$

- 3 Ruining the hard work you just did, create these two $-\mathbf{Darryl}_1$ vector arrows. (Don't change the mass hangers. Just the vectors.) You'll do this by just reversing the directions of both the vector arrows you initially created.

Leaving the $-\mathbf{Darryl}_1$ pointing at 180° , add the $-\mathbf{Darryl}_2$ in the usual tail-to-head fashion.

\mathbf{Larry}_3 is the sum of these two. Create the green \mathbf{Larry}_3 vector from the tail of \mathbf{Darryl}_1 to the head of \mathbf{Darryl}_2 .

Upload a file with your graphical representation of this vector addition. Label your three vectors \mathbf{Larry}_3 , $-\mathbf{Darryl}_1$, and $-\mathbf{Darryl}_2$.

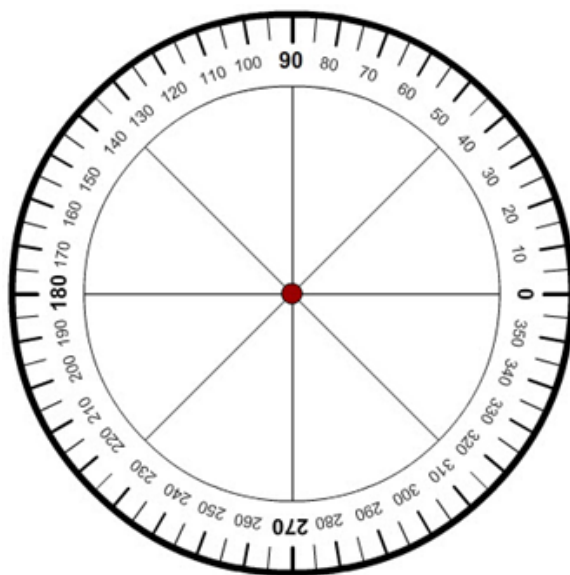


Figure 11: Larry and 2 Darryls

- 4 Record \mathbf{Larry}_3 (graphical) from the length and direction of your green \mathbf{Larry}_3 vector.
Move pulley 3 to the position indicated by the direction of \mathbf{Larry}_3 . You'll want to temporarily turn off the hangers to set the angle correctly.
Place the necessary mass on hanger 3 to produce the \mathbf{Larry}_3 force.
- 5 Record \mathbf{Larry}_3 (experimental).

Ta-da! I hope this has made vectors a little bit less mystifying.