

Image Formation by Plane and Spherical Mirrors

PURPOSE

- To become familiar with the nature of the images formed by plane and spherical mirrors.
- To learn to distinguish between real and virtual images.
- To discover the relationships among object position, image position, focal length, magnification, and the radius of curvature of plane, converging, and diverging mirrors.
- To become familiar with sign conventions and how and why they're used.

EQUIPMENT

Virtual Optical Bench

Pencil

SIMULATION AND TOOLS

Open the Virtual Optical Bench simulation to do this lab.

EXPLORE THE APPARATUS/THEORY

Open the Virtual Optical Bench simulation.

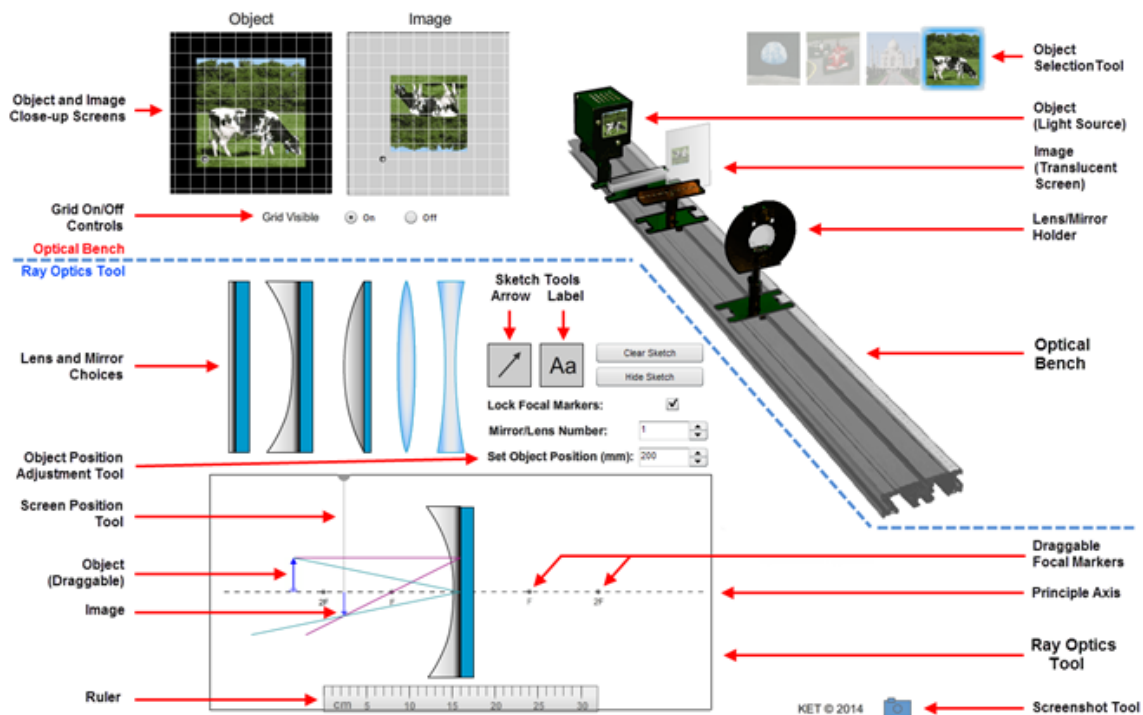


Figure 1: Optical Bench Apparatus

The virtual Optical Bench Apparatus simulates the formation of images by plane mirrors and by converging and diverging mirrors and lenses. We'll focus on mirrors in this lab.

Were it not for the development of eyes, there wouldn't be a lot to say about the interaction of light with reflective and refractive media except that it sometimes reflects and sometimes refracts, and sometimes does both. But because of the wondrous way that our eyes can take the light leaving an object and reconstruct a replica of that object on our retinas which can then inform the brain of the original object, we have developed the powerful study of optics.

Everyone is very familiar with the plane mirror and has had at least some experience with curved mirrors and lenses in a number of settings. From our extensive experience with our daily primping we feel very confident with the use of plane mirrors. But when we first try to use a car's rear-view mirrors, we find that we have a lot more to learn.

When you start up the Optical Bench Apparatus, it should look like Figure 1. A dashed line has been added in Figure 1 that separates two parts of the apparatus. Below the line is the Ray Optics Tool, where light rays are used to show the geometry involved in the formation of images. Above the dashed line is the Optical Bench and its object and image tools and screens. The Ray Optics Tool serves another important function: it controls the Optical Bench. Let's try it with the concave (converging) mirror.

Move your pointer over each of the five lenses and mirrors. The info box will identify each one and tell you how to switch choices. Click and drag the plane mirror until your pointer is over the concave mirror (below on the Ray Optics Tool) and release. The concave mirror should be replaced by the plane mirror. If not, try again. Remember, your pointer needs to be over the device being replaced when you release the button. Now swap them back. We want to work with the concave mirror.

To the left of the mirror are two vertical arrows. The one pointing upward with the small light blue drag handle on the bottom is the Object Arrow. It represents whatever object we're working with. As suggested by Figure 2, it emits light in all directions. Actually light is emitted in this way from every point on the object. We display only two (sometimes three) of these rays. Drag the Object Arrow from side to side as far as you can. You'll see that the rays are continually redrawn, and the other arrow, the Image Arrow, can move, resize and flip vertically.



Figure 2: Object Arrow

There's also action on the Optical Bench. Notice how the little box with the cow on the front moves in sync with the Object Arrow on the Ray Optics Tool. That's the Optical Bench's object. It's a slide illuminated from behind. Thus it emits light rays in all directions. Leave the object somewhere near the back end of the Optical Bench.

You've seen that the Image Arrow on the Ray Optics Tool moves, flips, and changes size. To let us see the image on the Optical Bench, we allow light from the object to illuminate a screen. To

form an image on the translucent screen, it must be moved to where the image is *located*. You'll examine that concept in detail soon.

The Screen Position Tool on the Ray Optics Tool is used to move the screen on the optical bench. It's a vertical gray line with a semicircular handle at one end. Drag it from side to side and notice how the screen moves on the Optical bench. Also notice that when it's to the left of the concave mirror a smudge of light appears on the screen.



Figure 3: Screen Position Tool

As you move the object, the smudge looks more or less like an inverted cow, depending on the location of the object. Leave the screen somewhere between the mirror and the object.

If you have for some reason changed the Mirror/Lens Number to something other than 1, change it to 1 now. You'll want to leave it there until you start to work with unknowns in section IIA.

Drag the object to the point labeled 2F on the left end of the ray tool. (You can also move the object with your left and right keyboard keys.) Drag the Screen Position Tool to the same 2F point. Ta da! A nicely focused, upside down cow appears on the frosted glass. An enlarged version also appears in the Image Close-up Screen next to the Object Close-up Screen. Each screen has a grid that can be moved around to make size measurements. Each grid square is 1 cm on a side. Also, try zooming in on the cow object and image on the bench. They should be side-by-side and equal in height. You can right click (Mac: CTRL click) as many as six times to zoom way in. "Show all" will take you back to 100%.

You can trade in your cow for a Taj Mahal, a fast red car, or the moon, depending on your mood. The text of this lab generally refers to the cow as the object until the last part of the lab where we'll trade it in for the Taj Mahal. "Upside-down cow" carries a more serious tone than "upside-down moon."

PROCEDURE

Please print the worksheet for this lab. You will need this sheet to record your data.

I. Plane Mirrors; Virtual and Real Images

We're going to look at the behaviors of our three mirrors to learn the nature of the images they form as well as the geometry behind the image formation. Then we'll look at the algebraic descriptions of this geometry. We'll start with the plane mirror.

Drag the plane mirror in place on the Ray Optics Tool. It should look something like Figure 4 minus a few added details.

The **law of reflection** states that when a ray of light reflects from a mirror the **angle of incidence** equals the **angle of reflection**.

The lower, green-colored rays illustrate this. A ray leaves the top of the Object Arrow, strikes the mirror at some point (its midpoint in Figure 4) and then reflects down to the left. The dashed line conveniently helps us see that the angles are indeed equal.

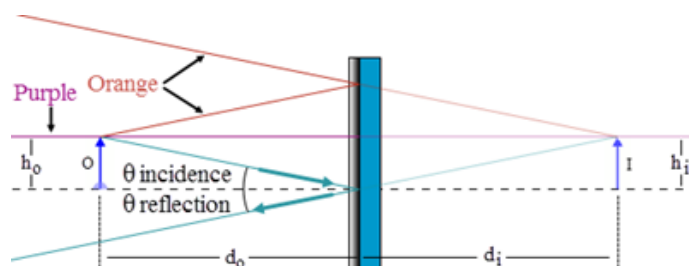


Figure 4: Image Formation by a Plane Mirror

There are two other ray pairs in Figure 4 that illustrate the same behavior. The orange rays at the top clearly show the same effect. But the purple rays in the middle are less obvious. A ray leaves the top of the object and hits the mirror normally, that is, at a right angle to the mirror. Thus it reflects straight back on itself. The angles of incidence and reflection are both zero in this case. Remember, there are many more rays from the object which hit the mirror. We just need at least two for our work.

Behind the mirror (on the right), we see the image. Its **height, h_i , is clearly equal to that of the object, h_o ,** and with the draggable ruler you can verify that **the image is as far behind the mirror, d_i , as the object is in front, d_o .** This becomes even more obvious when you move the object toward and away from the mirror. (We'll discuss the way that the light seems to bounce off the back surface of the mirror in the next section.)

- 1 As you drag the object toward the mirror, picture the equivalent activity of walking toward a bathroom or dressing room mirror. If you walked toward the mirror at 2 m/s, at what speed is your image moving toward the mirror?
- 2 At what speed is your image moving toward you? Remember, you're moving too.

We clearly see our image move toward and away from our body when we move toward and away from a plane mirror. But there's really nothing there. An image is not something that has any solid reality in the physical world like the object it mimics. If this is so, how can it have a location?

To understand the location of images, we first need to understand **how we locate actual objects** when we look at them. In Figure 5a we've added a pair of eyes. These belong to a person, seen from above, who is located between the object and the mirror. That is, you're looking down on the top of this person's head. This person is ignoring the mirror for now. Be sure to put yourself

in these figures. For the plane mirrors you can actually reenact events by standing in front of a bathroom mirror.

Note that the eyes are pointed toward the top of the object so that a different ray from the object enters each eye. Think about it. At any given instant you are actually just looking at one small region of space. For example, you can't look at a complete sheet of paper. Try it. Your eyes will start madly searching all over it. When you read this sentence you are only looking at a little bit at a time. The rest of it is handled very differently than the part you're actually focusing on. Consider how much work it would be to actually see all of a brick wall at once and have to keep actively seeing all of it to "keep it there." We actually see something more like "brickness." Sorry, I didn't mean to alarm you.

Where were we? The two little eyes were pointed at the top of the object so that they can let the two rays from just that part of the object reach your retina. Most of the time, your eyes are pointed at the same thing. Your eyes can't look at two different things at the same time. Instead, at a given instant, they both "agree" to focus their attention on one thing, the tip of the Object Arrow in this case. In addition, based on the amount the eyes have to turn relative to straight ahead, we compute the distance to the object and adjust the focus of the eye's lenses to properly focus the light. Amazing. You probably remember learning to do that by examining your feet when you were lying on your back as a baby.

Try it. Look at something far away. Now, focus on something close by. You can feel your muscles straining to turn your eyes away from their relaxed parallel alignment. (A good vacation spot always provides an ample supply of interesting things to look at with relaxed eyes. An ocean, for example!) Again, this is the **process of locating an object**, that is, assigning it a location.

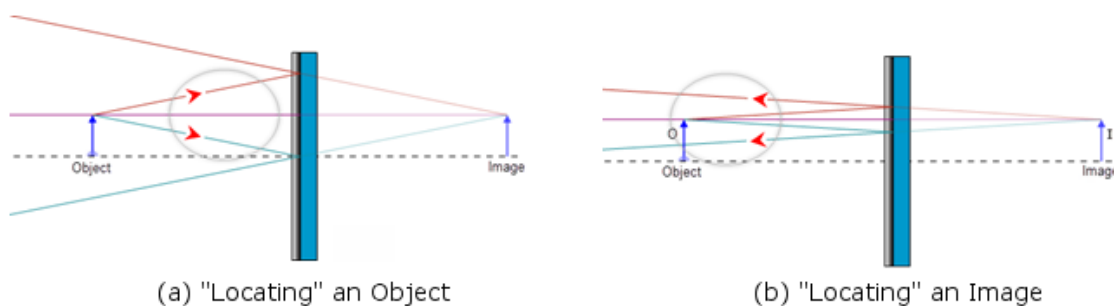


Figure 5: "Locating" an Object and an Image

In Figure 5b, the person has turned around and is now facing the mirror. Now light is allowed to reflect from the mirror before entering the eyes. (Magically passing through the person's head.) Note that a different pair of rays, from the many available, is used this time. In Figure 5b, the reflected rays entering the eyes look no different than the direct rays used in Figure 5a. They're all just rays of light coming from a specific point. The mind does the same triangulation. But this time it notices that your eyes don't have to point inward so much and concludes that the more parallel rays are coming from a source farther away and **behind the mirror**. All the information that's needed for the mind to conjure up an impression of the existence of the object is available.

We say that the person is now looking at (locating) an **image** of the object. When the rays entering

the eye are in any way redirected from their straight line path away from the object, we say that the observer is looking at an image. (Even mirages are images.) In this particular case, if you extrapolate back to the apparent source of the light, the image, you find that the rays never passed through that location (behind the mirror). We call this a **virtual image**. Virtual means “in effect, but not in reality.” Plane mirrors always form virtual images—the image will always be behind the mirror where the light never goes. And since there’s no light present at the image, you **can’t form a virtual image on a screen there**.

You’ll see that our Optical Bench is similarly limited. The Ray Optics Tool will always show where the image is located because it just uses geometry to locate it. But the Optical Bench realistically creates the image using light coming from the object. If no light strikes the screen, no image can be formed on it. Thus the screen can only display **real images**.

Let’s take a closer look at these two types of images. Replace the plane mirror with the converging (concave) mirror. Move the object to the 2F point. Move the Screen Position Tool to the 2F point, which is where the Image Arrow is located. You’ll see the **real image** on the screen of the Optical Bench and on the Image Close-up Screen. The light is actually hitting the screen and point by point creating the full image. Our Ray Optics Tool just follows a pair of rays from the top-most point on the object. But if you added rays from other points you’d find that they will produce their images at the proper places necessary to build up the full image.

Does this mean that you **have to** have a screen to see a real image? No.

Consider Figure 6 with the frosted image screen at the location of the inverted image shown. With the screen in place, the light arriving at the tip of the Image Arrow would be scattered in all directions. Some of this light would travel into the eyes. So the eyes would see light coming from the screen. If the tip of the arrow was pink, the eyes would see pink light coming from that point.

Now move the screen away. The light no longer scatters from the image screen. But if you were to put your eyes in the location shown in Figure 6, the rays will continue on into your eyes just as if there was an actual (pink) object at the tip of the inverted image shown. You would then see the virtual image and locate it sort of hanging in space at that point.

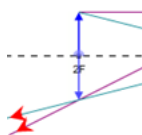


Figure 6: Seeing an Image Without Using a Screen

The images below illustrate a great example of this. In the first image you see a push pin at the bottom of a concave mirror. In the second image you see a second concave mirror with a hole in its center sitting on top of the first one. But this one’s mirrored surfaces faces downward. Note also that it has a circular hole in the top. An image of the push pin and part of the lower mirror is formed by the two mirrors. The image is created at the location of the hole in the top mirror. The push pin appears to be sitting on a mirror at that location. When you study corrective lenses later in this course, you’ll learn how they take light from an object at a location where you can’t see things clearly and produce an image of it where you can. For example, if you’re nearsighted,

you may only be able to see things clearly out to 1 meter from your face. Your lenses will take light from a distant mountain and form an image at approximately 1 meter from your eyes!



We've seen two different types of image—real and virtual. Let's summarize our key observations about them.

REAL VS. VIRTUAL IMAGES:

For **real images**, the light passes through the image and then diverges. We can put a screen at the location of the image and see it displayed there. Or we can allow the light diverging from the image to pass into our eyes which can then form an image (of an image) on our retinas.

For **virtual images** the light only appears to diverge from the image. The light doesn't actually pass through the image. And as a result a virtual image can never be displayed on a screen.

The Nature of an Image

Categorizing an image as real or virtual is a part of the process of determining the **nature of an image**. The nature of an image is determined based on three criteria.

To state the nature of the image you'll pick one from each of the following.

- a** Is the image **real** or **virtual**?
- b** Is the image **reduced in size**, the **same size**, or **enlarged in size** relative to the object.
- c** Is the image **upright** or **inverted** relative to the object?

- 3 Record the nature of the image formed by a plane mirror. Choose one for each category.
 - a Real or virtual? (real, virtual, no image)
 - b Size? (reduced, same size, enlarged)
 - c Orientation? (upright, inverted)

As noted earlier, for a plane mirror, the image is as far behind the mirror as the object is in front and the size of the image and object are always the same. One further measurement that will be introduced in more detail soon is the magnification, M . The magnification is the ratio of the image height, h_i , over the object height, h_o . So for a plane mirror this will always be equal to 1.

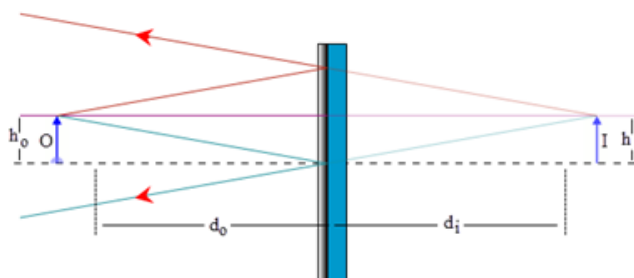


Figure 7: Object, Image Size, and Position for a Plane Mirror

II. Concave (Converging) Mirrors

- 1 The second mirror choice, which you've already seen, is a concave mirror. Drag it onto the Ray Optics Tool and then drag the object until the screen looks similar to Figure 8.

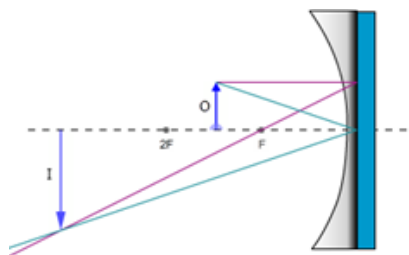


Figure 8: The Concave (Converging) Mirror

- 2 Both the object and the image are on the left side of the mirror. Drag the Screen Position Tool all the way to the right. Now start dragging it slowly to the left and watch the screen on the Optical Bench. Drag it just “through” the mirror and stop.
- 3 Notice what happened on the Image Close-up Screen at the top left of the lab screen. There's something fuzzy there. That's a very poor image of a cow. The problem is that it's not focused because the screen isn't where the image is located. Drag it a little more and you'll see the image begin to clear up both on the close-up screen and on the Optical Bench screen. Adjust the screen position until the image is well focused. Use the close-up screen to make these determinations.

4 What is the nature of this particular image? Check one for each category.

- a Real or virtual? (real, virtual, no image)
- b Size? (reduced, same size, enlarged)
- c Orientation? (upright, inverted)

In the first question asking you to make these choices, you were describing the “nature of the image formed by a plane mirror.” For any plane mirror, there’s exactly one answer for each category. This time the question was not so general. You were asked about “this particular image.” As you’ll soon see, the nature of the image produced by a spherical mirror changes according to the object’s distance from the mirror.

We’ll pay a lot of attention to the nature of images in this lab. It’s important. Here’s an example of why it matters.

- 5 Dentists use mirrors to clearly see what they’re doing when they’re working on your teeth. Would you want your dentist to use a mirror system arranged like the one on your screen now? What would be the advantage of this arrangement? What would be the disadvantage? Look at the Object and Image Close-up Screens.

A. Focal Length, f , Object Distance, d_o , and Image Distance, d_i

The nature of the images formed by a concave mirror is not as neat and tidy as what we found with plane mirrors. The nature of the image and its location depend on the particular curvature of the mirror and the location of the object relative to the mirror. As shown in Figure 9a, **the object distance, d_o , is the distance from the center of the reflective surface of the mirror out to the object, O.** Similarly, **the image, I, is located a distance d_i from the center of the reflective surface of the mirror.** (Just as with plane mirrors we draw the rays and make our distance measurements as if they reflect off the back surface of the mirror.)

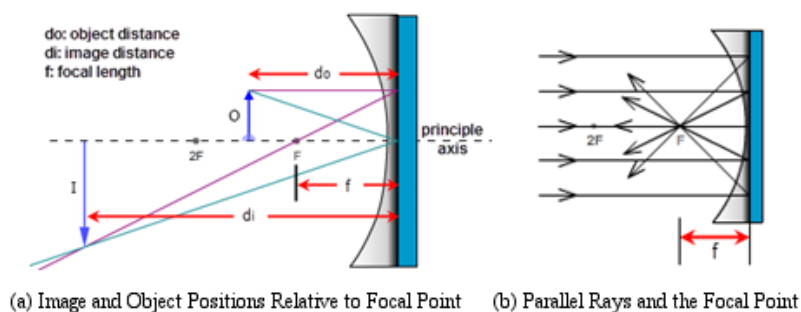


Figure 9: Image Formation by a Concave (Converging) Mirror

The property of the mirror that determines the nature of the image is its **focal length**, which is determined by the curvature of the mirror. The **focal point**, or **focus, F**, from the Latin word for “hearth,” is the point where rays of light traveling **parallel to the principle axis** of the mirror come together after reflecting from the mirror. See Figure 9b. (With convex mirrors the definition of the focal length is somewhat different as we’ll see.)

The focal length, f , is the distance along the principle axis from the focus to the center of the mirror.

- 1 We can find this experimentally with our Ray Optics Tool. Using the Object Position Adjustment Tool, (Set Object Position), adjust the object position to 120 mm. Notice that the **incident rays**, the two rays leaving the tip of the object arrow, are maybe 20° apart. The **reflected rays**, the two rays reflecting off the mirror, cross below the axis near the left end of the axis.
- 2 Click the “up” arrow on the Object Position Adjustment Tool tool until the object is at 280 mm. As you do this, notice how the angle between the incident rays becomes increasingly smaller and that the point where the reflected rays cross gets increasingly closer to the mirror and to the axis.
- 3 Repeat the preceding steps until you’re clear on that.
- 4 Now start clicking the “up” arrow on the Object Position Adjustment Tool for a while and watch the two pairs of rays. Stop at about 1 m.
- 5 What did you observe about the trend in the angle between the two incident rays? That is, what is the angle between them approaching?
- 6 Reduce the object distance back to 120 mm and notice where the reflected rays cross at the tip of the Image Arrow.
- 7 Begin slowly clicking to gradually increase the object distance and observe how this crossing point moves.
- 8 As you increase the object distance, the incident rays continued to get closer to parallel. What’s happening to the reflected rays? Where does their crossing point appear to be heading? Click and hold the object position tool until the object position reaches 3 m. Zoom in a couple of times for a close-up look.
- 9 What you’ve observed is the behavior of a concave spherical mirror. **Rays parallel to the principle axis of a concave spherical mirror reflect and pass through the focal point** as in Figure 9b. While you’re zoomed in, grab the ruler and measure the **focal length, f** , of the mirror #1 mirror. Measure from the back of the mirror where it’s attached to the bluish mounting surface, out to the **focal point, F** . It should be 7.5 cm. You can now right-click and select “show all” or “100%” to return to a normal magnification of the apparatus.

Incidentally, the reverse behavior is also true. If you place a light source, say a small bulb, at the focal point, the rays will reflect off the mirror parallel to the axis. This is the principle of a flashlight, searchlight, etc.

The focal length is not an arbitrary distance. A spherical mirror is actually a small section of sphere. Thus it has a certain **radius of curvature, R** . **The focal point is half way between the mirror and its center, C** . That is

$$f = \frac{R}{2}. \quad (1)$$

For convenience we label the center point $2F$.

- 10 Click through the five mirror numbers and notice how the curvature of the mirror determines the focal length.

From here on you'll work with one of these mirrors that has been selected for you. You'll load that mirror by setting the Mirror/Lens Number to an integer from 2 through 5. These are not focal length values, just numbers identifying four different mirrors. Each focal length is a whole number multiple of 5 mm (4.0 cm, 6.5 cm, etc.).

- 11 Find the mirror number selected for you in WebAssign and select the matching mirror in the apparatus.
- 12 Using the techniques you used with mirror #1, measure and record your mirror's focal length.
- 13 Using your focal length value and the ruler, drag the F and $2F$ icons on the left side of the mirror to their proper places on the Ray Optics Tool. You won't need the other pair for now. Use the "Lock Focal Markers" check box to lock them in place.

B. Images Formed by a Concave Mirror

With our plane mirror, we found that the image and object are always equidistant from the mirror and that the nature of the image is that it's always upright, virtual, and the same size as the object. We'd like to be able to determine all the same information about images formed by concave and convex mirrors. We'll develop methods for determining this information geometrically with drawings and algebraically with equations based on the geometry. With concave mirrors, there are a number of different outcomes all depending on the relative values of the focal length and the object distance. They're different in that the nature of the image is different for each. This is important if you want an enlarged (relative to the object) image, a reduced image, etc., for a given situation. Although there are several outcomes, actually five, there is a logical pattern. We specify these as cases I–V.

Yes, you do need to know these cases and be able to draw them!

Case I: Object distance, $d_o > 2f$ ($d_o > R$)

For case I, the object is at any distance greater than two times the focal length from the mirror. That is, the object is beyond the $2F$ point. Drag the object back and forth between $2F$ and the left end of the principle axis to see its range of motion. Also notice the range of motion of the image.

- 1 Move the object to about half-way between the $2F$ point and the left end of the axis. You'll see that the image is formed between the focal point, F , and $2F$ points.
- 2 Adjust the Screen Position Tool until you get a sharp image on the screens.

Here's how the geometry works to form an image for this case.

If we can find the image of one point on our object, all other points can be similarly found. We'll start with the tip of the object arrow. All rays leaving the tip of the object, after hitting the mirror, will pass through the matching point on the image and then continue. There is no limit to the number of rays that you could draw from the tip of the arrow to its image. And any two will do since all reflected rays converge at the same point.

If you picked another point on the object, say at the middle of the arrow, you'd find that the rays would converge at another point on the image. It would be at its midpoint. These rays are very busy!

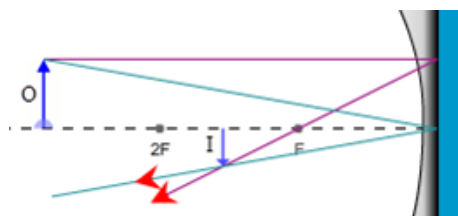


Figure 10: Eyes Locating a Real Image

Our apparatus draws what's called the **parallel ray in magenta**. This ray is easy to draw since it leaves the object traveling parallel to the axis and then reflects back through the focal point.

Our apparatus draws a second, **cyan** ray, which strikes the center of the mirror and reflects back at an equal angle below the axis. It's not reliable in hand drawings since an "equal angle" is not easy to draw. So we won't use it in our hand drawn figures. Instead we'll choose from two others for our hand drawings. Here are the three to choose from.

Ray 1. The **magenta, parallel ray** goes from the tip of the object, parallel to the principle axis, and then reflects back through the focal point.

Ray 2. The **green, focal ray** goes from the tip of the object, through the focal point, and then back parallel to the principle axis. Note that the mirror in Figure 11 should have a smaller radius of curvature—less flat—to produce this ray since the ray should travel along a radius. But adjusting the mirror shape to match different focal lengths is tricky. So we just use one standard mirror for all cases. Sorry.

Ray 3. The **orange, chief ray** goes through the center of curvature, C , (at $2F$) and then reflects back through C .

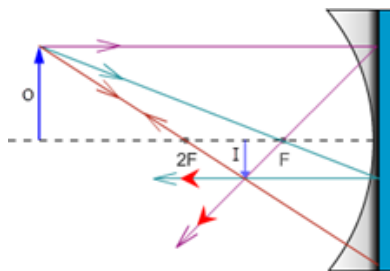
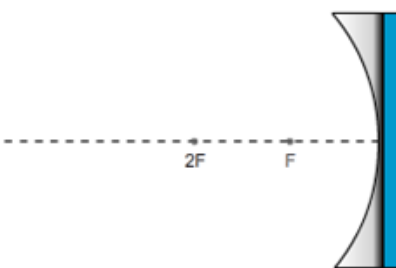


Figure 11: Three Rays Used in Locating Images with a Concave Mirror

- 3 Take a **Screenshot** of the Ray Optics Tool. Just include the object, image, and mirror. **Save the image as “mirrors_case_I.png”, and upload it.**
- 4 The figure provided in Sketch 1 contains a mirror and principle axis with the F and 2F points labeled. To this figure add the **object, image, and two rays to locate the image**. You have three rays to choose from as described and illustrated in Figure 11. Use Ray 1 and either Ray 2 or Ray 3. So your diagram will look just like Figure 11, but with one fewer rays.



Sketch 1: Case I $d_0 > 2f$

Are you clear on what these drawings mean? We want to know where the image is as well as its nature. We said earlier that when rays of light from an object are redirected before they enter your eyes, the light will appear to come from a different location. The three rays in Figure 11, along with all the others not shown, diverge from the object and then converge at the image. But they then diverge apart just as they do from an object. If any two of these diverging, reflected rays enter your eyes, you’ll be able locate the image. To emphasize this, you should also **include a pair of eyes at the end of your two rays just as we did in Figures 10 and 11.**

- 5 How about the nature of the image for case I? Check one for each category.
 - a Real or virtual? (real, virtual, no image)
 - b Size? (reduced, same size, enlarged)
 - c Orientation? (upright, inverted)
- 6 We want a reminder of how the object and image compared in size and orientation. The cow used as our object will never change. (Don’t feed it during the lab.) There is an enlarged image of it at the top left of the screen with the corresponding image next to it. **Take a Screenshot of the pair, save it as “mirrors_caseI_cows.png”, and upload it.**

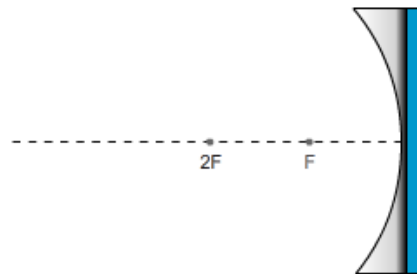
- 7 In case I, the object can be anywhere beyond a distance of $2f$ from the mirror. That’s a lot of territory. But it’s a specific range of territory—everywhere beyond $d_o = 2f$. The image also has a specific range of positions. **The image distance, d_i , is the distance from the image to the center of the mirror.** Drag the object back and forth, keeping outside the $2F$ point. How would you define the possible locations for the image?

Case II: Object Distance, $d_o = 2f$ ($d_o = R$)

We’ll repeat most of what was done in case I now for case II. The instructions will be repeated but streamlined.

In case I, the object and image both had a range of possible positions. In case II, the object can be at exactly one point—the $2F$ point. Not surprisingly the image has only one possible location.

- 1 Drag the object to the $2F$ point. (You can also use the cursor keys after clicking on the object.)
- 2 Adjust the Screen Position Tool until you get a sharp image on the screens.
- 3 Take a **Screenshot** of the Ray Optics Tool. Just include the object, image, and mirror. **Save the image as “mirrors_case_II.png”, and upload it.**
- 4 In Sketch 2, create your own ray diagram as you did with case I. Be sure to **include a pair of eyes at the end of your two rays.**



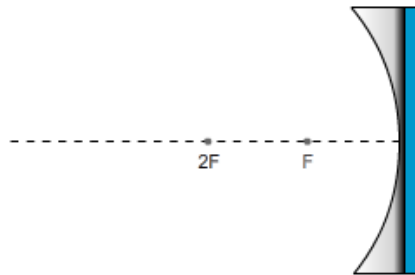
Sketch 2: Case II $d_o = 2f$

- 5 How about the nature of the image for case II? Check one for each category.
- a Real or virtual? (real, virtual, no image)
 - b Size? (reduced, same size, enlarged)
 - c Orientation? (upright, inverted)
- 6 Take a **Screenshot** of the object and image displays at the top left of the screen, save it as **“mirrors_caseII_cows.png”**, and upload it.
- 7 In case I, the object could be anywhere beyond a distance of $2f$ from the mirror. In case II, the object has no room to roam. Likewise the image has a fixed position. How would you define the possible locations for the image?

Case III: Object Distance, $f < d_o < 2f$

Now we return to a situation where the object has a range of positions. Drag the object back and forth between F and 2F. Notice the range of motion of the image.

- 1 Drag the object to about halfway between the F and 2F points. (You can also use the cursor keys.)
- 2 Adjust the Screen Position Tool until you get a sharp image on the screens.
- 3 Take a **Screenshot** of the Ray Optics Tool. Just include the object, image, and mirror. **Save the image as “mirrors_case_III.png”, and upload it.**
- 4 In Sketch 3, create your ray diagram. Be sure to **include a pair of eyes at the end of your two rays.**



Sketch 3: Case III $f < d_o < 2f$

- 5 How about the nature of the image for case III? Check one for each category.
 - a Real or virtual? (real, virtual, no image)
 - b Size? (reduced, same size, enlarged)
 - c Orientation? (upright, inverted)
- 6 Take a **Screenshot** of the object and image displays at the top left of the screen, save it as “**mirrors_caseIII_cows.png**”, and upload it.
- 7 In case I, the object could be anywhere beyond a distance of $2f$ from the mirror and the image was confined between F and 2F. In case III, the object again has a range of possible positions. How would you define the possible locations for the image?

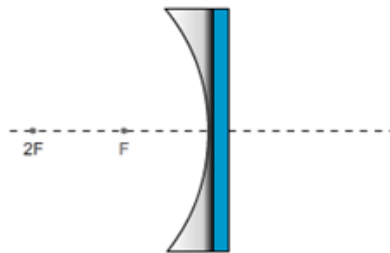
Case IV: Object Distance, $d_o = f$

Similar to case II, we again have a situation with just one position for the object. We'd also assume that we'd again have just one position for the image. As you'll see, that would in fact be wrong! How can that be?

- 1 Starting with the object at the 2F point, drag it slowly toward the focal point, F. Actually the cursor keys are better here. Notice how the rays reflected from the mirror behave. Keep moving until the object reaches the focal point. Then continue until it's about half-way between F and the mirror.

Before the object reaches the focal point the reflected rays are converging to the left of the mirror. As you get closer, they converge farther to the left and as a result the image is formed farther to the left.

- 2 After the object passes through F, where do the rays converge?
- 3 Drag the object back and forth through the focal point. What can you say about what must be happening to the reflected rays when the object is exactly at F?
- 4 Given that the (tip of) the image is formed where the reflected rays intersect, what can you say about the image?
- 5 Take a **Screenshot** of the relevant section of the Ray Optics Tool. **Save the image as “mirrors_case_IV.png”, and upload it.**
- 6 In the figure provided, create your ray diagram. Be sure to **include a pair of eyes at the end of your two rays.**



Sketch 4: Case IV $d_o = f$

- 7 How about the nature of the image for case IV?
- 8 Take a **Screenshot** of the object and image displays at the top left of the screen, save it as **“mirrors_caseIV_cows.png”**, and upload it.

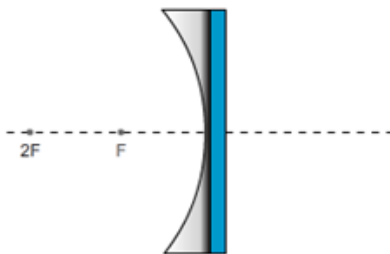
Case V: Object Distance, $d_o < f$

Now we return again to a situation where the object has a range of positions. Drag the object back and forth between F and the mirror. Notice the range of motion of the image.

- 1 Drag the object to about halfway between the F and the mirror. (Or use the cursor keys.) You can only get so close.
- 2 Adjust the Screen Position Tool until you get a sharp image on the screens. Hmm. That doesn't work so well. There is an image and you can move the screen to that point, but the image doesn't show up on the screens. You should be able to use what you've learned so far to make a clear statement about why you don't see an image on any screens. It will help to look at the geometry that produced your previous images. You might want to go back to your work with plane mirrors for insight. This is why you're asked to draw the eyes on all these figures. Discuss these three crucial points:

- a Why did we see images on the screens in cases I–III?

- b Why don't we see an image on the screens in this case (V)?
 - c Why then is there an image in this case and what must we do to see it?
- 3 Take a **Screenshot** of the relevant section of the Ray Optics Tool. **Save the image as "mirrors_case_V.png", and upload it.**
 - 4 In the figure provided in Sketch 5, create your ray diagram. Be sure to **include a pair of eyes at the end of your two rays.**



Sketch 5: Case V $d_0 < f$

- 5 How about the nature of the image for case V? Check one for each category.
 - a Real or virtual? (real, virtual, no image)
 - b Size? (reduced, same size, enlarged)
 - c Orientation? (upright, inverted)
- 6 Take a **Screenshot** of the enlarged object and image displays at the top left of the screen, save it as "**mirrors_caseV_cows.png**", and upload it.
- 7 In cases I and III, the object could travel between infinity and $2F$, and $2F$ and F respectively. The image locations were the reverse of this. In case V, the object again has a range of travel, this time between F and the mirror. How would you define the possible locations for the image?

III. Convex (Diverging) Mirrors

And now for something completely different. Switch to the convex, diverging mirror. Use the same mirror number as before. It will have the same focal length as the converging mirror, but as you'll see later, the sign will be negative.

- 1 Using your focal length value and the ruler, drag the F and $2F$ icons on the right side of the mirror to their proper places on the Ray Optics Tool. You won't need the left pair for this type of mirror. Use the "Lock Focal Markers" check box to lock them in place.

By bulging outward in the center rather than inward a convex spherical mirror has a very different effect than a concave mirror. Each type of mirror has a focal length and a radius of curvature, but

these two points, F and $2F$, are on opposite sides of these two types of mirrors. Figure 12 illustrates the difference in the behavior of these two types of mirrors.

You may have noticed that we have two names for each of these mirror types. One pair of names describes their shapes—*concave* and *convex*. But when we begin discussing their behaviors we often switch to the terms describing how they bend light—*converging* and *diverging*. One advantage of using the latter terms is that converging mirrors and lenses act similarly as do diverging mirrors and lenses. While concave mirrors act similarly to convex lenses! Not so good.

Let's quickly look at this. Put the object between F and $2F$. Now swap back and forth between the converging mirror and the converging lens. (The one that bulges in the middle.) They work exactly alike. The images are just on different sides of the devices. Note that their case behaviors are identical, too. So when you understand converging mirrors, you get converging lenses for free! Try the diverging devices, and you'll see the same 2 for 1 special.

When you study lenses, you'll see that we will use the F and $2F$ points on both sides of the lenses. Can you see why?

So just how do converging and diverging devices bend light?

Converge means to come together. Rays parallel to the principle axis of converging devices bend light *toward the focal point* as shown in Figure 12a (repeat of Figure 9b).

Diverge means to separate, split, or spread. Rays parallel to the principle axis of diverging devices bend light *away from the focal point* as shown in Figure 12b.

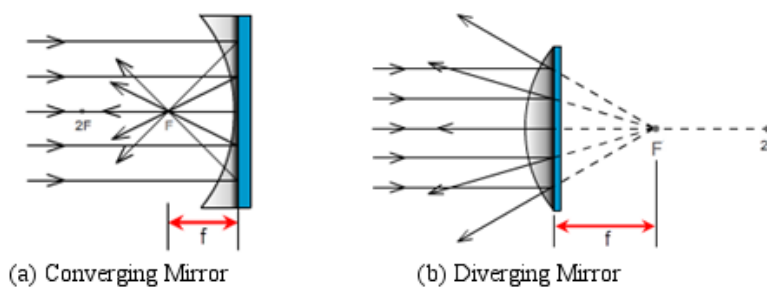


Figure 12: Converging and Diverging Mirrors

Let's now compare the way that these two types of mirrors produce images. In the box below for the concave mirror, you see what you learned earlier about three rays often used to locate images produced by a concave mirror.

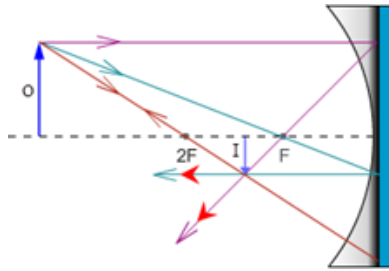


Figure 11: Three Rays Used in Locating Images with a Concave (Converging) Mirror

Ray 1. The **parallel ray** goes from the tip of the object, parallel to the principle axis, and then reflects back through the focal point.

Ray 2. The **green, focal ray** goes from the tip of the object, through the focal point, and then back parallel to the principle axis.

Ray 3. The **orange, chief ray** goes through the center of curvature, C , (at $2F$) and then reflects back through C .

- 2 Using the diagram for a diverging mirror in Figure 13, complete the following sentences describing the matching results for a diverging mirror.

Ray 1. The **parallel ray** goes from the tip of the object, parallel to the principle axis, and then reflects back _____.

Ray 2. The **green, focal ray** goes from the tip of the object, toward the focal point, and then reflects _____.

Ray 3. The **orange, chief ray** goes toward the center of curvature, C , (at $2F$) and then reflects _____.

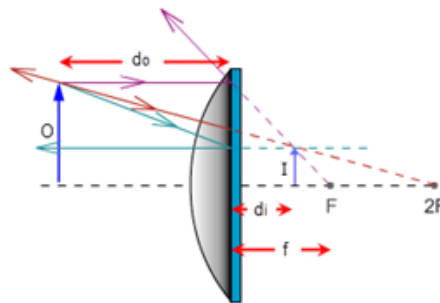
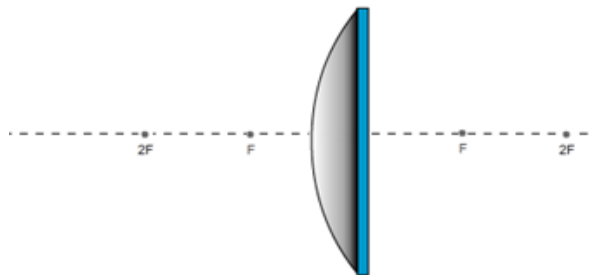


Figure 13: Three Rays Used in Locating Images with a Convex (Diverging) Mirror

- 3 Start with the object at the left end of the Ray Optics Tool and drag it slowly toward the mirror. Notice how the rays reflected from the mirror behave. Keep moving until the object is as close to the mirror as possible. Not a lot of variety!

- 4 Drag the object back to a point somewhere between $2F$ and F . (There's nothing special about this region. It just provides a good size figure.)
- 5 Take a **Screenshot** of the relevant section of the Ray Optics Tool. Save the image as “**mirrors_diverging.png**”, and upload it.
- 6 In the figure provided in Sketch 6, create your own ray diagram. Be sure to **include a pair of eyes at the end of your two rays**.



Sketch 6: Diverging Mirror

- 7 How about the nature of the image for a diverging mirror? Check one for each category.
 - a Real or virtual? (real, virtual, no image)
 - b Size? (reduced, same size, enlarged)
 - c Orientation? (upright, inverted)
- 8 Take a **Screenshot** of the enlarged object and image displays at the top left of the screen, save it as “**mirrors_diverging_cows.png**”, and upload it.
- 9 Describe the position or range of positions available to the image in this case.

Learning about the various image cases has a practical purpose. We need different image types for different situations. Sometimes we need an enlarged image and sometimes we need it reduced. But we also care about the orientation of the image. An upside down image in some cases could cause big problems. In other cases it would not be a problem.

- 10 Explore these considerations by matching up the image cases in Table 1 with the applications listed below. Each letter choice gets used exactly once.

- a Telescope
- b Flashlight, headlights, searchlights
- c Dentist's mirror
- d Shoplifting surveillance mirror
- e Photocopier in normal copy mode
- f Photocopier in enlarge mode

		Application Name
Converging Mirror	Case I	
	Case II	
	Case III	
	Case IV	
	Case V	
Diverging Mirror		

Table 1: Matching Image Cases to Typical Applications

IV. Equations and Sign Conventions for Spherical Mirrors

From our diagrams summarized in Figure 14, we can clearly see that the focal length and object distance geometrically determine the nature and location of any image. It should be very simple to produce equations to describe this geometry algebraically, and it is. There's one problem, however. For cases I-III, you get one pair of equations, then a slightly different pair for case V, and another slightly different pair for the diverging mirror. If we look at lenses while we're at it, we find a similar situation.

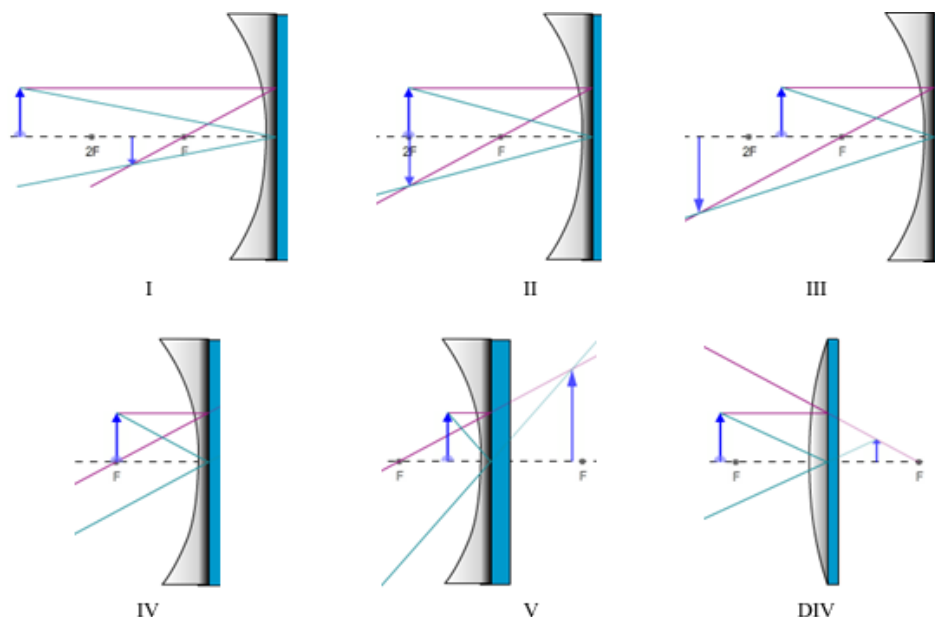


Figure 14: All Image Cases for Spherical Mirrors

We could just memorize several pairs of equations, but the equations only differ by the signs of the different terms. Further examination reveals that the different signs are connected to the different natures of the images. So since we already need to know about the natures of the images it makes sense to just use a pair of equations and a set of **sign conventions** that adjust the signs of f , d_o , d_i , h_o , h_i , and M according to the situation. What follows is the system that we'll use.

A. Sign Conventions for Mirrors and Thin Lenses for Use with the Mirror or Thin Lens Equation and the Magnification Equation

The following two equations can be used to describe any of the five converging mirror or lens cases as well as diverging mirror or lenses. The equations will always take the same form but the signs of the values inserted into the equations are determined by the sign conventions. For example, if an image is formed 12 cm behind a converging mirror, you'll insert a value of -12 cm for d_i . And if you were solving for that particular image distance based on a given f and d_o , the resulting d_i will be found to have a negative value.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \quad \text{Mirror or Thin Lens Equation} \quad (2)$$

$$M = -\frac{d_i}{d_o} = \frac{h_i}{h_o} \quad \text{Magnification Equation} \quad (3)$$

SIGN CONVENTIONS:

Focal Length, f (mirror or lens)

$+f$: Converging Device

$-f$: Diverging Device

Object Distance, d_o

$+d_o$: Object is on the side of the device from which the light arrives. (Real Object)

$-d_o$: Object is *not* on the side of the device from which the light arrives. (Virtual Object*)

Image Distance, d_i

$+d_i$: Image on the side of the device where light travels after reflection or refraction. (Real Image)

$-d_i$: Image not on the side of the device where light travels after reflection or refraction. (Virtual Image)

Magnification, M , and image height, h_i

$+M$, $+h_i$: Image is upright relative to the object.

$-M$, $-h_i$: Image is inverted relative to the object.

The **magnification**, M , is a measure of the degree of enlargement or reduction of the image relative to the object. It is a ratio of the images height, h_i , to the object height, h_o , and thus a dimensionless, signed, number.

CAUTION:

Be careful not to rephrase these conventions. They are easily broken. For example,

“Image not on the side of the device where light travels” can’t be translated into

“Image on the side of the device where light comes from” because of differences in lenses and mirrors.

* When more than one device (mirror or lens) is present, one device will form an image which becomes the object for the second device. If that *virtual object* is on the “wrong” side of the second device, that is, not on the side from which the light actually arrives, this *virtual object’s* object distance is negative.

In Figure 15, the image, I, formed by the large mirror could become the object for either of the two smaller mirrors shown. For mirror #1 the initial image acts as a real object since rays actually diverge away from it and then arrive at the mirror. For mirror #2 the initial image is behind the second mirror and acts as a virtual object.

We will not see virtual objects in this lab or consider them any further.

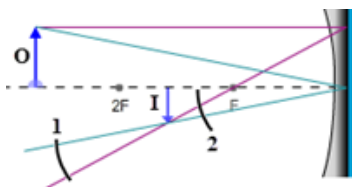


Figure 15: Virtual and Real Objects

EXAMPLE OF THE USE OF EQUATIONS AND SIGN CONVENTIONS:

If an object is placed 15.0 cm in front of a convex mirror and an image is formed 5.0 cm behind it, what must be the mirror’s focal length? If the image is 2.5 cm tall, what is the magnification and what is the height of the object?

Solution:

Known:

- | | |
|-------------------------|--|
| $d_o = +15.0$ cm | The object is placed in front of the mirror where the light comes from. |
| $d_i = -5.0$ cm | The image is behind the mirror, which is not where the light travels after reflection. |
| $h_i = +2.5$ cm | We know that an object in front of a diverging mirror produces upright images. |
| $f = ?, M = ?, h_o = ?$ | The signs of these unknowns will be determined as described in the following note. |

A NOTE ABOUT SIGNS:

When substituting the numeric values for these six variables into an equation, you must provide the signs for each variable. Do not insert an assumed sign for the unknown.

When you solve for an unknown, the signs you provide for the knowns, along with the algebra, will produce the correct sign for the unknown being solved for.

The focal length: If we use Equation 2 to solve for f , the algebra will produce a negative value for f . Solving Equation 2 for f , and substituting in our given values gives us:

$$f = \frac{d_i d_o}{d_i + d_o} = \frac{-5.0 \text{ cm} \times 15.0 \text{ cm}}{-5.0 \text{ cm} + 15.0 \text{ cm}} = \frac{-75 \text{ cm}^2}{10.0 \text{ cm}} = -7.5 \text{ cm}.$$

The negative sign indicates that this is a diverging mirror.

This focal length was selected for this example because it's the default for our apparatus. With your apparatus, select mirror #1. Drag and drop the diverging mirror onto the the mirror or lens currently at the center of the Ray Optics Tool. Measure the focal length to see that it is indeed -7.5 cm. That is, 7.5 cm behind the mirror.

Move the object to 15 cm in front of (to the left of) the mirror. You'll see that it's at the 2F point which makes sense.

As you'd expect, the image can't be displayed on the screens since it's a virtual image. Measure, with the Ray Optics Tool, to confirm that the image is 5.0 cm behind the mirror.

The magnification: Equation 3 describes the relative sizes of the object and image.

$$M = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$

We can solve the left side of the magnification equation for M to find the magnification. Note the negative value inserted for d_i from our sign conventions.

$$M = -\frac{d_i}{d_o} = -\frac{-5.0 \text{ cm}}{15.0 \text{ cm}} = 0.33$$

The positive sign corresponds to upright images formed by a diverging mirror. The fact that the absolute value of M is less than one reflects the reduced size of the image.

We can't measure the image size since we can only measure images formed on the screen. But, using the Ray Optics Tool, you should be able to see that the image is about 1/3 the height of the object.

The height of the object: We can solve the right side of the magnification equation for h_o .

$$h_o = -h_i \times \frac{d_o}{d_i} = -(2.5 \text{ cm}) \times \frac{15.0 \text{ cm}}{-5.0 \text{ cm}} = 7.5 \text{ cm}$$

This is three times the image height and positive as expected.

Notice that the negative sign before 2.5 cm was part of the original equation. h_i is positive for this case.

So by including the proper signs on the known values, we get a result with signs that can be interpreted to identify the type of mirror, object, image, etc. Knowing the five cases for a convex device and the single case for a diverging device lets us provide the proper signs and interpret the ones we compute.

- Let's practice. Insert a plus or minus sign, or N/A for each choice in Table 2. Assume that the object is in front of the mirror in each case. Only real objects are being considered. You might want to use the sign conventions chart and Figure 14. Notice how they say similar things in different ways. It's probably instructive to do this one column at a time.

		f	d_o	d_i	M	h_i
Converging Mirror	Case I					
	Case II					
	Case III					
	Case IV					
	Case V					
Diverging Mirror						

Table 2: Matching Variable Signs to Image Cases

The magnification can be tricky since its sign indicates one type of information while its value relative to 1.0 indicates another. This second consideration is due to the fact that it's a ratio of h_i/h_o , which can be >1 , <1 , or $=1$.

- What does the sign of the magnification indicate about whether an image is upright or inverted?
 - A positive sign indicates that the image is _____.
 - A negative sign indicates that the image is _____.
- The absolute value of the magnification can be less than 1.0, equal to 1.0 or greater than 1.0. Ex. 0.5, 1.0, or 3.2.

What does each of these indicate about the image?

- a A magnification whose absolute value is less than 1.0 indicates that an image is _____.
 - b A magnification whose absolute value is equal to 1.0 indicates that an image is _____.
 - c A magnification whose absolute value is greater than 1.0 indicates that an image is _____.
- 4 A magnification of +3.2 means that the image is _____ times the size of the object and _____.
- 5 A magnification of -0.32 means that the image is _____ times the size of the object and _____.

B. Determining Magnification and Image Location with the Mirror and Magnification Equations

The example problem we've just explored should give you a good idea of the skills you need to work with image formation using mirrors. We'll finish this lab by doing similar explorations of each image case and both mirror types.

- 1 Use the same mirror number selected for you in WebAssign for Part II.A.
- 2 Record the focal length you found for your mirror in Part II.A., step 12.

For each of these cases, you'll find a table similar to the sample table below, which you will complete using the following procedure.

Experimental	d_o (cm)	—
	d_i (cm)	—
	h_o (cm)	—
	h_i (cm)	—
	$M_{\text{experimental}} = h_i/h_o$	—
Calculated	d_i (cm)	—
	$M_{\text{calculated}} = -d_i/d_o$	—
% Difference	% diff. in d_i	—
	% diff. in M	—

Table 3: Sample Data Table (Mirror type and case will be specified here.)

PROCEDURE TO COMPLETE TABLE FOR EACH CASE:

Each data table has three sections—**experimental**, **calculated**, and **% difference**.

a Experimental data:

- i** Select a representative object position for the case being examined and then measure and record d_o and d_i , using the centimeter ruler on the Ray Optics Tool.
- ii** To measure h_o and h_i use the **draggable** grid on the Object and Image Close-up Screens as described below.
- iii** Also calculate and record h_i/h_o as your experimental value for the magnification. In the case of virtual images you won't have an experimental value for h_i or M since you can't measure h_i . In these cases you'll find "N/A" already inserted into the table.

Let's use the Taj Mahal for the rest of the lab since it's a more regular shape than a cow. The central building, the tomb, is at the center of four 130-foot-tall minarets, one at each corner of a square. The two minarets in the foreground, of course, look taller. For our **object**, we'll use the shorter-looking minaret to the left of the tomb. This minaret is on the right in the Object picture in Figure 16. The small, inverted minaret shown in the Image picture is its **image**. (Remember, the image is inverted and reversed left to right.)

We won't use the actual 130-ft heights of the minarets. We'll use the measured sizes of the objects and images on our screens instead. Both **Screenshots** in Figure 16 were taken with double magnification. (Zoomed in twice.) You might want to do the same. You can click and drag the grid to line up the fine scale lines with the pictures. Each grid square is 1.0 cm on a side, so the fine scale markings are 0.10 cm apart. **The minaret (object) looks to be about 2.3 cm tall to the top of its spire. Let's all use that value for the experimental object height, h_o .**

For the 7.5-cm converging mirror used to make these images, the image in the right figure looks to be about 1.4 cm tall for this case. You'll make and record your own h_i measurement for your assigned mirror.

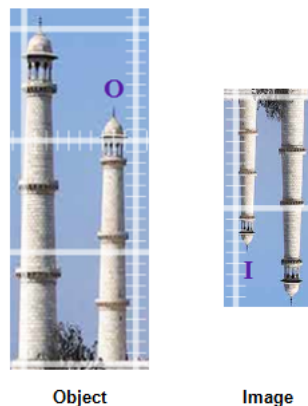


Figure 16: Object and Image Views of Taj Mahal

PROCEDURE TO COMPLETE TABLE FOR EACH CASE (CONTINUED):

b Calculated values:

- i** Use your measured d_o , and your assigned focal length in Equation 2 to calculate the image distance, d_i .
- ii** Use your measured d_o , your calculated d_i , and Equation 3 to compute your calculated M .

c Percent differences:

- i** Finally find the percent differences between your **experimental** and **calculated** values for d_i , and M . This won't be possible in the case of virtual images. Again, you'll find N/A in the table for these cases.

To clarify the procedure, here's an example. For case I, which involves an object beyond the 2F point, you'll select and record an object distance and place the object at that point. You'll then adjust the Screen Position Tool to move the Optical Bench screen to be in focus and record its location as d_i . You'll then zoom in two times and use the magnified image grid to measure the height of the image. (We're assuming that the object height is 2.3 cm.) With these you'll calculate $M_{\text{experimental}}$. You'll then calculate d_i , and $M_{\text{calculated}}$ using Equations 2 and 3. Finally, you'll compare these calculated values to the experimental values for the same quantities using percent difference. In some cases you won't be able to make some of these measurements or calculations because the virtual images can't be measured.

- d** Show all the calculations for $M_{\text{experimental}}$, d_i calculated, $M_{\text{calculated}}$ and the two percent-age differences in the space beside the table. Remember that the percentage difference is computed as follows.

$$\% \text{ difference} = \frac{|\text{experimental} - \text{calculated}|}{(|\text{experimental}| + |\text{calculated}|)/2} \quad (4)$$

- 3** For the concave mirror—case I: $d_o > 2f (R)$, do the following using the above procedure.
- a–c** Complete Table 3.
 - d** Show your calculations of $M_{\text{experimental}}$, $d_{i \text{ calculated}}$, $M_{\text{calculated}}$, and both percent differences.
- 4** For the concave mirror—case II: $d_o = 2f (R)$, do the following using the above procedure.
- a–c** Complete Table 4.
 - d** Show your calculations of $M_{\text{experimental}}$, $d_{i \text{ calculated}}$, $M_{\text{calculated}}$, and both percent differences.
- 5** For the concave mirror—case III: $f < d_o < 2f (R)$, do the following using the above procedure. (Be sure your object image does not go outside of the window.)
- a–c** Complete Table 5.
 - d** Show your calculations of $M_{\text{experimental}}$, $d_{i \text{ calculated}}$, $M_{\text{calculated}}$, and both percent differences.
- 6** For the concave mirror—case IV: $d_o = f$, do the following using the above procedure.
- a–c** Complete Table 6.
 - d** No calculations can be done for this case.
- 7** For the concave mirror—case V: $d_o < f$, do the following using the above procedure. (Be sure your object image does not go outside of the window.)
- a–c** Complete Table 7.
 - d** Show your calculations of $d_{i \text{ calculated}}$, $M_{\text{calculated}}$, and the percent difference in d_i .
- 8** For the convex mirror, do the following using the above procedure.
- a–c** Complete Table 8.
 - d** Show your calculations of $d_{i \text{ calculated}}$, $M_{\text{calculated}}$, and the percent difference in d_i .