## KET Physics Lab Guide for WebAssign 2014

Welcome to the KET Physics lab program. The labs you'll be doing have been designed to teach you good lab technique while giving you direct experience with the phenomena you're studying in the course. You're also encouraged to use the virtual lab apparatus outside of the regular guided lab program to help you answer your own questions.

Hopefully you'll enjoy working with this virtual apparatus. The 21-piece apparatus collection was designed and developed at Kentucky Educational Television (KET) by Nathan Pinney, Brian Vincent, and Tim Martin, and Chuck Duncan. The labs, that is, the lab activities, were written by Chuck Duncan. Work continues on both fronts.

The virtual labs are currently available for use by high schools or colleges and universities at two different websites.

- The home of the KET Virtual Labs is www.virtuallabs.ket.org/physics.

To work with the labs on that site requires that your school sign up for a school license. Instructions for obtaining a license are found at that site by clicking the Free Trial tab.

- The labs are also available at www.webassign.net. The WebAssign system allows students to submit their results including data, calculations, graphs, sketches, etc., electronically. With this system most grading is done automatically. Students are also able to request assistance from their teacher while working on the labs on WebAssign. Your teacher will provide further information about accessing the labs on that site.

A video introduction for each piece of apparatus is available at http://virtuallabs.ket.org/physics/apparatus/
To access the video for the apparatus you'll be working with, just click on its image on that page. On the page that opens you'll see "Watch: Video Overview." Click that link to view the video overview. These do not include sound.

You'll use this Lab Guide in three ways.

1. You may be asked to use it as a lesson to complete before you do your first lab. There are a number of activities contained in this guide and for best results you should complete all of them. Plan on two to three hours for this.
2. You'll use it as a reference when you're asked to do certain things in the lab. For example you'll be asked to find percentage errors. That's explained in section 1.4, and you'll learn (or review) that calculation when you study that section.
3. You'll use it to learn how to use the Grapher software which is supplied as part of the lab program. Instructions on the use of the built in Sketch tools are also provided.

Grapher can be found at two locations.
a. If you're accessing the labs at the KET site, look for the Grapher icon at the end of the lab apparatus list on the Labs tab. Click it to take you to the Grapher page. Click on the larger icon (screen image) there to open Grapher.
When you're using Grapher along with a piece of virtual apparatus you'll need for each one to be in its own browser tab. This generally means that you'll right-click on the second Grapher icon to open it into a new tab and then click on the Labs tab again to find and open the apparatus you'll be working with. This again involves clicking on two icons. (The order of opening these doesn't matter.)


In WebAssign, any lab that uses Grapher will include a link which will open Grapher in a new tab.

There's a lot of information in this document. Don't panic. You'll come to understand it better as the course progresses.

## WILL LAB WORK BE ON THE TEST?

Your Physics course will teach you many concepts and techniques that will help you understand and work with the physical world. Your regular course instruction, your labs, and your homework work together to that end. Depending on the course you're in you may or may not be asked specific lab-related questions on your tests. But you'll be enhancing your understanding of course concepts in the lab, and this is very likely to have a positive effect on our test results.

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## Are you a PC or a Mac?

The lab apparatus and Grapher software will run on either a Windows or Mac computer. But the two sometimes handle some things differently. Here's one to be on the lookout for.
Windows computers have at least two mouse buttons. - a left and a right, and sometimes a middle one.
If you're told to click on something, this means to move your pointer over it and click with the left button.
We'll say right-click when we mean to use the right button. A right-click usually brings up a menu of choices to select from.

Some Macintosh computers have just one button. It's equivalent to a regular (left) click. A ctrl-click (holding down the control key and clicking) is equivalent to a right-click.

## 1. ERRORS AND UnCERTAINTY

As you'll discover in this course, physics is mostly about events, that is, things that happen.

- A baseball is hit and travels along an arc or along the ground and the fielder has to get to the right place at the right time to catch it.
- Billiard balls collide at various angles and you can somehow predict which way they'll go.
- The tea I'm making right now will melt most of a glass of ice if I don't wait five minutes before I pour it in.
- The length of time between phone recharges depends on the screen brightness, amount of use, etc.
- The damaging effect of light depends on how much of various wavelengths are involved.

We'll describe these events in terms of distances, angles, times, temperatures, current, voltage, wavelength and other measurable quantities. In the lab we'll find out how these and other quantities are related by qualitative observations ("it's a very high fly ball"), measurement ("the tea has cooled a bit, I'll need less ice"), and mathematical description ("power $\times$ time $=$ energy, so maybe I'll get to talk longer between charges if I get a bigger battery.")
Let's look at some of the tools of the trade that help us make sure that our lab work is up to code. We'll begin with a the concepts of uncertainty and error.
Uncertainty is inherent in all measurement. So you can't eliminate it. But the measurements you make in the laboratory should be made as accurately as possible. The numbers you record, with appropriate units, express the amounts of your measurements as well as their uncertainties (error). This lab manual will introduce you to the techniques we'll use to accomplish this. The methods of error analysis and even the definition of key terms found in the literature vary considerably. In future courses you may be introduced to more sophisticated error analysis.

Scientific Error is not a bad thing. When we refer to error it's just another term for uncertainty.

## 1.1 - Types of Error

Our measurements are limited in two ways - instrument limitations and random uncertainties or random errors.
Instrument limitations: Our measuring devices have limits to how finely they can divide up a quantity. The finest graduation on a meter stick is a millimeter. We can use it to measure to within some fraction of a millimeter, but not beyond. We would say that 1 millimeter is the resolution of a standard meter stick. If we removed the millimeter marks, leaving centimeter graduations we would decrease its resolution to one centimeter. Thus we would reduce its resolution by a factor of ten.

Random (indeterminate) errors: When you measure the length of an object with a meter stick you have to judge when the object and meter stick are aligned properly and which points on the meter stick align with each end of the object. If you made the same measurement several times you'd get slightly different results, ranging by some amount either side of some central value. This type of random variation is inherent in any measurement, but the average of many such measurements would be a good indication of the "correct" value.

What about human error? This is not actually a scientific term. The two types of limitations mentioned above are normal parts of any scientific measurement and we can deal with them in standard ways as you'll see. But "I didn't realize that I was using a yardstick and not a meter stick" is just a mistake. We'd deal with it by repeating the experiment with the proper tool. So just eliminate that term "human error" from your scientific vocabulary.

## 1.2 - Precision and Accuracy

To its extreme embarrassment, precision's definition varies according to the context of its use. In physics we generally use the term in two ways.
\#1. Precision is the degree of agreement among several measurements of the same quantity; the closeness of repeated measurements to the same value; the reliability or reproducibility of a measurement.

Note that precision is unrelated to the correctness of a measurement. If you shoot at a target several times and all the arrows are close together you're a precise shooter. If the arrows are again tightly grouped but actually in the wrong target, you're a precise but inaccurate shooter.
You may see precision expressed like this: $2.04 \pm .05 \mathrm{~m}$. This would indicate that your average value for several measurements was 2.04 and that the measurements were spread over a range between 1.99 m and 2.09 m . So you feel
confident that any subsequent measurement would fall into that range. A less precise measurement of the same object might be written as $2.04 \pm .1 \mathrm{~m}$. And $2.04 \pm .02 \mathrm{~m}$ would be more precise.
\#2. Precision is the measure of how exactly a measurement is made; the number of significant digits to which it can be measured reliably.

According to a plastic ruler a nickel is about 2.10 cm in diameter. If I used a device called a Vernier caliper I can measure more precisely. I might find the diameter to be 2.096 cm . The extra digit indicates the greater precision of the instrument being used.

In our work the term usually refers to the second definition. We also use the term resolution when using this definition. A plastic school protractor might measure angles to a few tenths of a degree, while a sophisticated scientific tool might measure to thousandths of a degree. Similarly a photo taken by a satellite passing over Mars might have a resolution of 10 meters. This means that the light from a $10 \mathrm{~m} \times 10 \mathrm{~m}$ area on Mars illuminates just one pixel on the digital camera's light-gathering chip. So if the light from that area is reddish on average, the pixel will be reddish. A similar effect is found with your eyes. As you approach a distant object the image spreads over a larger area on your retina, increasing the resolution of the image, letting you resolve smaller objects.

Ex. "I see beach." "Now I see sand." "Now I see that the sand is multicolor and jagged." "Help me up!"
Closely related to precision is accuracy. Accuracy is the degree of closeness of a measured or calculated quantity to its actual (true) value; the extent to which the results of a calculation or the readings of an instrument approach the true values of the calculated or measured quantities.

The results of calculations or a measurement can be accurate but not precise, precise but not accurate, neither, or both.
This may be better understood by an analogy. Consider several attempts by a marksman to hit a bull's eye. If the bullets all hit in a tight pattern we'd say that the shooting is very precise. This would be true even if the tight cluster is far from the bull's eye.

If that tight cluster was centered on the bull's eye we'd say that the shooting was both precise (definition 1) and accurate.

If the cluster was not so tight, but still centered on the bull's eye, we would say that the shooting was accurate, but not precise.
We won't be doing any shooting in the lab, but we will be making multiple measurements of quantities. If our repeated measurement of a quantity are nearly the same (precise) and approximately equal to the "correct" value (accurate), we'd say that our measurements are both precise and accurate. A result is called valid if it is both accurate and precise.

Incidentally, the shooter whose bullets hit in a tight cluster away from the bull's eye would be said to have a systematic error in his shooting. What would the judges in that event have to say about this? "Sorry buddy, we don't forgive the 'human error' of the choice of an inadequate weapon, or the improper adjustment or use of the gun sight. You lose."

Systematic errors are not related to uncertainty. They indicate that you've done something incorrectly and need to correct the source of the error and retake your data. For example, if you were weighing liquids in a beaker and forgot to subtract the weight of the beaker all your values would be off by an amount equal to the weight of the beaker.

## 1.3 - Uncertainty of Measurement

All scientific measurements are made by comparison to some accepted standard. The length of the spine of a book is compared to markings on a standard meter stick; the time for a ball to fall some distance is compared to the seconds ticked off on a stopwatch, etc.

When I asked a visitor to my house how tall he was he responded " 2 meters, 4 centimeters." Was this tall Scandinavian exactly 2.04 m tall? Who knows? He was just really tall and that's all that we were curious about.

But as a scientific measurement 2.04 meters means something very specific. If we recorded a measurement of the distance that a rock had fallen as 2.04 m , this value would mean that we were confident that it fell some distance between 2.00 m and 2.10 m and that the $4(\mathrm{~cm})$ is our best guess of the final digit. We will always use this system when we take data. Here's the rule:

## You should always measure as many digits as you can with certainty and then estimate one more digit.

For a typical length measurement in the lab we can measure to a much higher resolution than 2.04 m . A meter stick has millimeter markings on it. Let's measure the length of the spine of a book with the section of meter stick shown below. Our measured length would be the difference in the right and left measurements. The left measurement is between 6.1 cm and 6.2 cm . We need to estimate one more digit. Let's say 6.14 cm . Our uncertainty is due to our inability to resolve tenths of millimeters. It's also compounded by the curve of the spine and the separation between the book and the ruler. The other end seems to be between 12.7 and 12.8 . We'll estimate 12.72 cm .

We'll record the book's length as $(12.72-6.14) \mathrm{cm}=6.58 \mathrm{~cm}$ or .0658 m . Note that our answer gives the magnitude of our measurement as well as an indication of the uncertainty of our measurement. We are confident to within .1 cm and estimate another digit. (This alignment of the instrument is really pretty terrible. Perhaps this is an example of that favorite scapegoat human error!)


Figure 1: Measuring Length with a Meterstick
There are many more sophisticated ways of expressing and calculating uncertainty. You may encounter them if you continue in your scientific studies. Let's quickly look at one that you may be familiar with before we proceed. (We won't use this method.)

Returning to Lars' height, we could go one step further and state how certain we are about that last digit. As you know, measuring a person's height is pretty uncertain given the squishiness of hair and the difficulty of getting the mark on the wall just right. So on a good day, you'd be happy with getting within a centimeter. So you might give Lars' height as $2.04 \pm .01 \mathrm{~m}$. (value $\pm$ uncertainty)

Beyond this lies statistical analysis using standard deviations and partial derivatives for the calculus students among you. Again, we won't use these methods.

## Significant Digits (Figures)

In the two examples above the uncertainty of our measurements (hundredths ( 2.04 m ) and ten-thousandths of a meter $(6.58 \mathrm{~cm}))$ has been clearly indicated by the digits we recorded. This is standard practice and you'll always use this system in your lab work. All you do is write down every digit you measure, including an estimated digit and you're done. These measured digits, including the estimated digits are referred to as significant digits. There is a bit of confusion involving zeroes since a zero can indicate either a measured value (significant) or be placeholders (nonsignificant.) So let's look at how we deal with them.

## Identifying significant digits

## Rule 1. Non-zero digits are always significant.

Ex. 3.562 m - all four digits are significant.
Rule 2. Embedded zeroes are always significant.
Ex. 2.05 cm - you estimated the 5 hundredths of a centimeter after measuring the 0 tenths of a centimeter with certainty, so the zero is certainly significant. This measurement has three significant digits.

## Rule 3. Leading zeroes are never significant.

Ex. 0.0526 m - just three digits are significant.
Leading zeroes are just place-holder digits. If you measured 0.0526 m in centimeters you'd write 5.26 cm . The units used should not change the number of significant digits. This number has three significant digits.

Rule 4. Trailing zeroes are always significant. This is where it gets tricky. Look at the following three scenarios to see why. The last two show how to deal with two different situations. A convention is used to prevent ambiguity.

Ex. 4.570 m - the zero comes to the right of a decimal point
You wouldn't include the zero unless you measured the last digit as a zero since 4.57 m is the same amount as 4.570 m . So you only needed to include the zero to indicate your estimate of the number of millimeters (0) after measuring the certain value of 7 cm .

Ex. 4570 m - all four digits are significant since you measured three and estimated the zero.
In this case you measured the 4 , then the 5 , then the 7 , and then estimated that the last digit was zero. It's no different than, say, 4572. The last digit was the estimated digit and just happened to be zero.

Ex. $457 \theta$ m, but you measured the same object but with a less precise instrument.
So it's the same size measurement as the previous example, but the seven was the estimated digit. And the zero is just a placeholder. So you really have only three significant digits.
Well rules are rules. If that trailing zero is not significant, you have to get rid of it. Here's how. If you need some trailing zeroes for placeholders, use scientific notation or metric prefixes instead.
Solution 1. $4.57 \times 10^{3} \mathrm{~m}$
Solution 2. 4.57 km
Got that?

- If you record 4570 m , then by convention, we know that the zero was a real measurement - your estimated digit.
- If you didn't really measure the final zero, that is, the 7 was your estimate, by convention, you can't include the zero. So you use scientific notation or the equivalent - metric prefixes - to indicate the location of the decimal point.


## Significant figures in the result of a calculation

In addition to making measurements to the proper precision we also have to attend to significant digits when making calculations. There are basically two situations to be concerned with. Here's an example of each. Calculate what you think the answers are. We'll return to these questions shortly.

Ex. 1. Suppose you measured the inner diameter of a pipe to be 2.522 cm and the length to be 47.2 m . What would you list as its internal volume? $\qquad$ $\mathrm{m}^{3}$.

Ex. 2. A truck that weighs 32,175 pounds when empty is used to carry a satellite to the launch pad. If the satellite is known to weigh $2,164.015$ pounds what will the truck weigh when loaded? $\qquad$ pounds

## Multiplication, division, roots and powers

After any of these operations, round the result off to the fewest significant digits possessed by any of the numbers in the calculation.

Ex. $2.53 \times 1.4 \rightarrow 3.542$ on the calculator
1.4 has the fewest significant digits (2). So we use 2 significant digits in our answer

Answer: 3.5

## Addition and subtraction

Before adding or subtracting, round each number to the uncertainty of the least certain number. Then add or subtract and keep the common uncertainty.

Ex. $2.53+1.4 \rightarrow 2.5+1.4$

$$
\begin{array}{lll}
2.53 & \rightarrow 2.5 \\
\underline{1.4} & \rightarrow \frac{1.4}{3.9}
\end{array}
$$

Notice that in the first case ( $\times, \div$, etc.) , we were counting sig. figs. In the second we were rounding to the least certainty. We won't discuss why this happens here. Just be aware of that key difference.

Now re-try example 1 above. Note that there's more than one calculation to make. Just do the rounding at the end of the calculation. $\quad\left(\mathrm{V}=\pi \mathrm{r}^{2} \mathrm{~h}=\pi(2.522 / 2 \mathrm{~cm})^{2}\left(4.72 \times 10^{3} \mathrm{~cm}\right)=23578.8 \mathrm{~cm}^{2}\right.$,
But, since this is multiplication, we just get 3 digits so $V=2.36 \times 10^{4} \mathrm{~cm}^{2}$

Now re-try example 2 above. $\quad(32,175+2,164.015) \mathrm{lb} .=32,175 \rightarrow 32,175$

$$
\underline{2,164.015} \rightarrow \quad \underline{2,164} \text { 34,339 lbs. }
$$

## Rounding

In both cases above we needed to round numbers, which involved removing trailing digits. There are different conventions for doing this. Here's how we'll do it.

Suppose we want to round to $\mathbf{n}$ significant digits.
Rule 1: If the digit immediately to the right of the $n^{\text {th }}$ digit is less than 5 , the number is rounded down.
Ex1. Rounding 2.434 to 2 significant digits becomes 2.4 after dropping 34 .
Rule 2: If the digit immediately to the right of the $n^{\text {th }}$ digit is greater than $\mathbf{5}$, the number is rounded up.
Ex2. Rounding 2.464 to 2 significant digits becomes 2.5 after dropping 64.
Rule 3: If the digit immediately to the right of the $n^{\text {th }}$ digit is $\mathbf{5}$ and there are non-zero digits after the $\mathbf{5}$, the number is rounded up.

Ex3. Rounding 2.454 to 2 significant digits becomes 2.5 after dropping 54.
Rule 4: If the digit immediately to the right of the $n^{\text {th }}$ digit is $\mathbf{5}$ and there are no subsequent non-zero digits, round the number in whichever direction leaves the $\mathrm{n}^{\text {th }}$ digit even.

Ex4a. Rounding 2.450 to 2 significant digits becomes 2.4 after dropping 50.
The result was left as 2.4 since 2.450 is closer to 2.4 than 2.6.
Ex4b. Rounding 2.550 to 2 significant digits becomes 2.6 after dropping 50.
The result was increased to 2.6 since 2.550 is closer to 2.6 than 2.4.

## 1.4 - Percentage Error and Percentage Difference

In the lab we will often try to determine the value of some "known" or theoretical quantity - the speed of light in a vacuum, the wavelength of light given off by an electrical discharge through a certain gas, or the acceleration of an object falling without friction. One way of comparing our experimental result with the theoretical value is calculating a percentage error between the theoretical and experimental value.

The error between the theoretical and experimental values is just the absolute value of their difference.

$$
\text { Error }=\mid \text { experimental }- \text { theoretical } \mid
$$

We can get a better measure of the significance of our error by comparing it to the theoretical target value. We can do this by calculating a percentage error.

$$
\text { Percentage Error }=\frac{\mid \text { experimental }- \text { theoretical } \mid}{\mid \text { theoretical } \mid} \times 100 \%
$$

Ex. The accepted value for the acceleration due to gravity is $9.82 \mathrm{~m} / \mathrm{s}^{2}$ at a certain location. In the lab a student measures it as $9.91 \mathrm{~m} / \mathrm{s}^{2}$.

$$
\begin{gathered}
\text { Error }=|9.82-9.91| \mathrm{m} / \mathrm{s}^{2}=.09 \mathrm{~m} / \mathrm{s}^{2} \\
\text { Percentage Error }=\frac{.09 \mathrm{~m} / \mathrm{s}^{2}}{9.82 \mathrm{~m} / \mathrm{s}^{2}} \times 100 \%=.916 \%
\end{gathered}
$$

The error doesn't mean much by itself. But by comparing it to the theoretical value of 9.82 we give it context. Calculating the percentage error tells us how big the error was relative to the amount of the measurement. A $1 \%$ error would mean that the student was off by 1 part in 100 , so $.916 \%$ is a bit less than that.

Sometimes we will want to compare two values for the same quantity when neither is known to be more certain than the other - the distance to a distant wall found with a meter stick and by sound reflection for example. In this case we can calculate a percentage difference.

$$
\text { Percentage Difference }=\frac{\mid \text { measurement }_{2}-\text { measurement }_{1} \mid}{\mid \text { average } \mid} \times 100 \%
$$

Ex. The speed of sound is determined in two different ways, once by measuring the time for a sound to bounce off a nearby building, and then by setting up standing waves in a tube. The two results are $318 \mathrm{~m} / \mathrm{s}$, and $340.6 \mathrm{~m} / \mathrm{s}$. While the second is measured to greater precision, we have no way of knowing how accurate either result is.

$$
\text { Percentage Difference }=\frac{|318-340.6|}{\left|\frac{318+340.6}{2}\right|} \times 100 \%=6.86 \%
$$

## 2 - Graphical Analysis; Working with Grapher

Important note - As you work through this section you need to create all the graphs described using Grapher. You'll be making your own graphs in the lab soon and uploading them as part of your lab reports. There is more information about using Grapher in section 3 .

## 2.1 - Graphs and Equations

In sections $2.1-2.15$ many examples of the use of Grapher will be illustrated. It is essential that you do every example as it's presented. You'll be using all these techniques frequently this year, so it's crucial that you learn the process now. The instructions below would also guide you in creating hand-drawn graphs.

Much of what we do in physics involves finding relations between quantities. These relations are best described using mathematical models - equations that relate the quantities. E.g. Speed $=$ distance/time. Probably the most well-known and important such mathematical relation involves pizza. Years of study in the laboratory has determined that doubling the diameter of a circular pizza quadruples its area. The inability of consumers to unravel this relation, thus allowing them to understand the pricing, is surely the reason why pizza is generally sold in a circular rather than square shape. (Well, there is the difficulty of tossing a square pizza crust.) Consider this NCLB exam question;):

Pizza A is 6" in diameter and costs $\$ 4.67$. Pizza B is $9 "$ in diameter and costs $\$ 6.21$
a) Pizza $A$ is the better buy.
b) Pizza B is the better buy.
c) Both prices are equivalent.
d) Pizza B is bigger so it's a better buy.
e) It depends on the number of slices.

In my experience answer (e) is the universal answer. Let's get to the bottom of this.
The key question is "how is the area related to the diameter?" A mathematical model, an equation summarizing the relationship, is what we need. Here's how we'd go about it.

Step 1. Go buy a bunch of pizzas. Check.
Step 2. I've already done the heavy lifting for you. Here's the data I took as I consumed the, err, "apparatus."

| Pizza | Diameter (in) | Area $\left(\mathrm{in}^{2}\right)$ |
| :---: | :---: | :---: |
| 1 | 2.11 | 3.14 |
| 2 | 3.94 | 12.6 |
| 3 | 5.82 | 28.3 |
| 4 | 9.15 | 63.6 |
| 5 | 10.02 | 78.5 |
| 6 | 12.25 | 113.1 |
| 7 | 13.84 | 154.0 |
| 8 | 16.01 | 201.1 |
| 9 | 20.14 | 314.2 |

By convention, the independent variable is listed on the left and the dependent variable is listed on the right. In this case, determining which is which is a bit iffy. In most cases it's clearer - the distance a car travels varies with time so time is the independent variable and the distance follows. Since we want to think about how the area depends on the diameter we'll consider the diameter to be the independent variable. You vary it and the area follows.
In a controlled experiment, which is our goal, any other potential variables must be held constant.

Step 3. Enter your area vs. diameter data in Grapher using the instructions below.
Note that that's dependent, $\mathbf{y}$ vs. independent, $\mathbf{x}$. Got that? This can be confusing. You'll put the variable that you're controlling, the independent variable, in the first column, and the variable that changes according to the relationship under study, the dependent variable, in the second column. When you use the "vs." terminology it's dependent vs. independent.

| $X$ | $Y$ |
| :--- | :--- |
| 2.11 | 3.14 |
| 3.94 | 12.6 |

Figure 2: y vs. x This is just by convention.
Open Grapher.
Click anywhere other than in the data table and then click in the first cell. Type 2.11
There are two ways to efficiently move from cell to cell while entering data.

- Click the Tab key after entering each value to fill in one row at a time.
- Click the Enter key after entering each value to fill in one column at a time.

Let's try it both ways.

- Click Tab and type 3.14 in the y-column. Click Tab again to return to the x-column. Continue in this way for the first 5 rows of data.
- Tab if necessary to get to the x-column in the sixth row. Type 12.25 and hit Return.

You're still in the x-column, but down a row. Enter 13.84 and continue to complete the $x$-column.

- Click in the first empty cell in the y-column and begin finishing that column using Return after each entry.

It's always up to you how to enter the data. Some find that when one column has a succession of easy numbers like 5 , $10,15,20$, using the Enter key to quickly move through the data is quickest. Suit yourself.

Step 4. Plot the graph.
For this data, the process is easy. Click the Auto button. The data will be plotted using Grapher's suggested axis ranges. The graph should look pretty good with no adjustments necessary.

The graph below is what you want for our finished product. Your graph shouldn't look much like this yet. Here's a description of what you want, followed by how to achieve it.


Figure 3: Pizza Area vs. Diameter
The dependent variable is plotted on the vertical (ordinate) and the independent variable is plotted on the horizontal (abscissa).

The graph should

- include a title that describes the experiment. You can also include other identifiers like figure numbers, the lab name, etc.
- fill the allotted space.
- be properly scaled. The scale of each axis should be uniform and linear.
- start at zero on each axis. Occasionally we'll deviate from this, but with care.
- include axes labeled with the name and unit for the quantity plotted.
- include carefully plotted data points surrounded by point protectors. (Large circles in our case.) To change the size of the point protectors, right click on the graph and choose small or large points. (Grapher's point protectors are of fixed size of no statistical relevance. They just make the location of the data points visible. Maybe in a later version...)
a) If the data is non-linear, draw a smooth curve indicating the tendency of the data as shown above. (In some cases you won't be able to do this with Grapher, but you can print out a copy and draw the smooth curve by hand if necessary.) For non-linear data you will usually need to linearize it to find the correct relationship as discussed below.
b) If the data appears to be linear, include a line of best fit indicating the tendency of the data. The data points should never be connected by straight line segments.
c) It is not important to hit any of the data points exactly. This includes the $(\mathbf{0}, \mathbf{0})$ point. The line or curve is a sort of averaging of the data.
d) When determining the slope of a linear graph by hand, include a pair of points on the line used in making the calculation. These should not be data points unless they happen to be on the line.
e) Do not do calculations on your graph.

Here's how we'll achieve this finished look (with the exception of the curve. More on that later.)
a) Click in the text box at the top of the graph that says "Title" by default. You can edit this text to create your title. Usually this title indicates what's plotted in "Ordinate vs. Abscissa" format. But you can add Figure numbers, lab name etc. You can click in this text area and drag the title around as needed.
b) Because you clicked the Auto scale button your graph should have been plotted so as to fill the space available. This system isn't foolproof but it works pretty well. It will do nicely with this data set.
c) If either axis doesn't start at zero, or needs some other adjustment, click the Man(ual) scale button. (Fig. 4a.) Enter $X_{\max }, X_{\min }, Y_{\max }, Y_{\min }$ values (Fig 4b.) to produce graph scales starting at 0,0 , and ending with values that lets the data fill most of the graph. The $X_{\max }$, and $Y_{\max }$ values should be chosen to make the graph easily readable. Clicking "Submit" will re-plot the graph with your new settings.


Figure 4a


Figure 4b
d) Click Close.
e) Your data is best summarized by your smooth curve or line of best fit. So once you create a line or curve, the data is no longer of any importance. To find the slope of a line by hand, pick two points on the line, as far apart as possible, and calculate the $\Delta y / \Delta x$. The slope should include units.
f) Your graph is probably much taller than the one shown in Figure 3. It's often convenient to reduce its height. For example you may be asked to print out a graph and attach it to your lab report. A smaller graph is usually preferred. You can easily reduce the graph to one half or one third of its initial height.
Try this. Click in the check boxes beside " 2 " and then " 3 " to see how this works. That's how Figure 3 was produced.


Step 5. Determine an equation relating area and diameter. We'll examine that process now.

## Creating an Equation by Linearizing Data; Using RMSE

You can often determine the relation between your variables and create an equation describing your data by a process called linearization. This is probably new to most students and it takes a bit of practice. The idea is to pick one variable and change each value in the same way - square each one, take the square root of each one, etc. If a plot using the new altered data is linear you can then write an equation, in $y=m x+b$ format, to describe your data.

For example if you found that a plot of $y$ vs. $x^{2}$ was linear, then $y=m x^{2}+b$ describes the relation between $y$ and $x$. The slope and y -intercept of your graph can then be used for $m$ and $b$ in the equation and you're all set. We'll see how to do this with an example and learn to use the Linear Fit tool in the process. The finished product should look like Figure 5.


Figure 5: Pizza Area vs. Diameter Squared

The "Graphs" tool allows you to plot a second and third graph of your data after calculating modified $x$ or $y$-values. You can choose to take first or second derivatives, natural logs, or squares of your $y$-values. You can also square the $x$-values. Here's what to do.


Figure 6: Graphs Tool
a. Add graphs \#2 and \#3 by clicking the check boxes below the data table. (Figure 6.)
b. Click the Graphs button to display the choices of the modifications to do for your second graph. Let's try them all. You're looking for a choice that will make Graph 2 linear. The default selection is $Y^{\prime}$, the derivative of $y$ with respect to $x$. (Don't panic you'll learn about that one later and no calculus is required.)

Note: After making each choice discussed below, click "Auto" to reformat the graph to display all your data.
You now have a graph of $y^{\prime}$ vs. $x$. It's not too bad but let's move on. $Y^{\prime \prime}$ is already plotted by default in Graph 3. Click its Auto button. This one's a mess.

Back to Graph 2, try $Y^{2}$. (Don't forget Auto.) Not good. Try the natural $\log$ of $y, \operatorname{Ln}(Y)$. Not good.
OK, now try $X^{2}$. This looks very good. Jump back to $Y^{\prime}$. Nope, $X^{2}$ looks best. There are endless choices of functions but all the ones we need are among these choices or can be generated by hand.

So let's go with the $X^{2}$ graph. Graph 1 was a plot of area vs. diameter. Graph 2 is a plot of the area (ordinate) vs. the diameter squared (abscissa). Your points should now appear to be in a straight line that goes through the origin.
c. Click in Graph 2. Click the Linear Fit checkbox. The line should now pass through your point protector circles. Notice that some are centered more above the line and some more below. This is the nature of a line of best fit. You'll see graphs in the lab that look almost linear but are not so randomly arranged. This indicates that the graph might not be linear.

Drag across several of the data points on your graph. You'll see that a different line is created. The line of best fit is generated for just the points you've selected. Now click anywhere in the graph. Since there is no area selected Grapher assumes that you mean to use all the data.
d. Grapher will automatically calculate the slope and y-intercept of the line of best fit using a least-squares fit calculation. It will also display the RMSE value. The RMSE (root mean square error) value provides a measure of the average distance of your data points from the line of best fit.

$$
\begin{aligned}
& y=m x+b \text { RMSE: } 2.3247 \\
& m(\text { SSope })=0.777 \\
& b(Y \text {-intercept })=0.394
\end{aligned}
$$

Important note: If you move your pointer over each graph you'll see that the linear fit box values will change to go with the graph that your pointer is in. Be careful to place your pointer in the correct graph! No clicking is required.

Let's see what the RMSE can tell us about our y-intercept. With your mouse pointer near the origin of graph, zoom in once. Notice how close your graph comes to passing right through the origin. It's very close. Is it close enough to say, within the experimental uncertainty of this data, that the $y$-intercept could be zero? If it is, then you could say that the area of your pizza is directly proportional to the square of its diameter and drop the small y-intercept value. That is, is the area proportional to the square of the diameter or is it proportional to the diameter plus or minus a little nibble? Zoom back out.
e. Write the equation in $y=m x+b$ form. Where,
$y$ : ordinate value, area (Area) $x$ : abscissa value, diameter ${ }^{2}\left(d^{2}\right) \quad m$ : slope $b$ : y-intercept
You should never use the general variables $\boldsymbol{y}$ and $\boldsymbol{x}$ in your equations. Always substitute the name or first letter of each variable. With typical values from this data shown in the best fit data box above,

$$
A=\left(.78 d^{2}+.39\right)
$$

But the y-intercept (.39) is quite a bit less than the RMSE (2.32), so the y-intercept is closer to the line than the uncertainty of your data you could say that it's essentially zero. So, there is no nibble! So our final equation is

$$
A=.78 d^{2} \quad \text { Not } y=.78 x^{2}
$$

We've linearized our data to determine the relationship between the area and the diameter. We didn't find a linear relationship between area and diameter. We found a linear relationship between area and diameter squared. And in this case, the data indicated that this was not just a linear relationship but a direction proportion.
We'll return to this process in a bit but we need to look at another important tool you'll be using first.

## 2.1 - Interpreting Graphs

In the pizza example we initially found this relation:

$$
A=.78 d^{2}+.39
$$

Let's look at the two terms $m$ and $b$. Calculating $4 \times \mathrm{m}=4 \times .78$ gives you 3.12 which looks a lot like $\pi$. Recalling that the area and diameter of a circle are related by

$$
A=\pi r^{2}=\frac{\pi d^{2}}{4}
$$

explains where the .78 came from. It's approximately $\pi / 4$.
What about b , the $y$-intercept? Clearly a pizza of zero-diameter should have zero area. This tiny bit of pizza (. $39 \mathrm{in}^{2}$ ) is a result of the uncertainty in measurement of the pizza. We can't just ignore our data, but as discussed earlier, it is within the range of error in our data so we can safely disregard it. As we'll see soon, this y-intercept will often have real physical meaning.
What we've found here is a common proportional relation - a quadratic proportion. While there are a huge number of possible relations, we'll find only a handful in elementary physics. In some of our labs we'll be using a graphical approach to find or verify these relations, so let's learn to recognize the ones that are likely to come up.

### 2.1.1 - No Relation

The time for a satellite to orbit a massive body is referred to as its period, $\boldsymbol{T}$. This time depends on surprisingly few factors. How about the mass of the satellite? Click the Clear button to empty your data table. (It may be hiding behind the Graphs tool.) Enter the period vs. mass data from Figure 7 into Grapher. (The Return method is much better than the Tab method here.) Be sure to click the Auto button.

Auto didn't work so well this time. Our graph doesn't start at 0,0 . Let's use Manual scaling. Click "Man" beside the top graph. Adjust as needed to produce Graph 7. You could use a different Ymax value if you like.

This data certainly seems linear, so turn on Linear Fit. Note the slope value in the Linear Fit box. The slope reflects how the dependent variable, T varies with the independent variable, m . How does T vary relative with m ?

It doesn't! Although we do find a straight-line graph, the graph has a slope of zero, indicating that the period does not change with mass. There is no relation between the mass and the period. (This surprises nearly everyone.)

| Mass <br> $(\mathrm{kg})$ | Period <br> $(\mathrm{min})$ |
| :---: | :---: |
| 200 | 90 |
| 400 | 90 |
| 600 | 90 |
| 800 | 90 |
| 1000 | 90 |
| 1200 | 90 |



Figure 7: Period vs. Mass

Incidentally no relation is often a great result. Proving that things are not related is just as important as proving that they are related. This is not the same as a null result where an expected result fails to show up. The Michelson-Morley experiment (1881) was designed to detect our velocity relative to the luminiferous aether which was the proposed medium through which light travels. Sound waves, water waves, etc., all travel through some medium which is actually what "waves." Michelson received the Nobel prize (1907) for physics for this failure to measure our velocity through this medium, thus proving its non-existence and setting the stage for Einstein's special theory of relativity.

### 2.1.2 - Linear Relation

An elastic object is one that stretches or compresses in a uniform way (up to a point called the elastic limit) as increasing amounts of force are applied to it. A spring is a good example. A rubber band is not so good an example as its elasticity changes as it's stretched by increasing amounts.

In Figure 10 a spring from the Simple Harmonic Motion lab is shown hanging beside a meterstick which measures its initial length to be about 30.0 cm .

When a 100 gram object is attached to the spring's lower end it will exert a downward force of .980 newtons $(.980 \mathrm{~N})$ on the spring. The mass will vibrate for a while. Once it comes to rest, the spring will have a new length of about 35.2 cm . As successive weights are added the spring stretches uniformly as shown in the figure.

The table in Figure 11 (using data from a similar experiment) indicates this change in a different spring's length as eight different weights are added.


Figure 8: Stretch

| Weight <br> $(\mathrm{N})$ | Length <br> $(\mathrm{cm})$ |  |
| :---: | :---: | :--- |
| 0.0 | 12.8 |  |
| 0.5 | 14.8 |  |
| 1.0 | 16.2 |  |
| 1.5 | 17.7 |  |
| 2.0 | 19.9 |  |
| 2.5 | 21.2 |  |
| 3.0 | 22.5 |  |
| 3.5 | 24.4 |  |
| 4.0 | 26.6 |  |



Figure 9: Length vs. Weight

The spring is initially 12.8 cm long and increases steadily in length as weight is added. When we plot length vs. weight we get a straight line indicating a linear relation between length and weight. In a linear relation the dependent variable changes by a fixed amount for a fixed change in the independent variable. That is, if adding 2 N of weight increased the length by 20 cm , adding another 2 N will increase it by 20 cm more. Note the stair step pattern in Figure 9 .

The graph indicates this relation by rising at a constant rate; that is, it has a constant slope. In our example, a slope of $3.337 \mathrm{~cm} / \mathrm{N}$ (units added) indicates that the length of the spring increased by about 3.337 cm for each newton of weight added. You'd want to use $3.3 \mathrm{~cm} / \mathrm{N}$ in your lab results since our weight and length data were to 2 and 3 significant digits respectively. So we're limited to the lesser number of digits when dividing to calculate the slope.

Note that the slope can also be negative for a linear relation. For example an employee's salary vs. time might be defined by a rising straight line while the distance to her assigned parking space could decrease yearly resulting in a falling linear relation.

The $\mathbf{y}$-intercept also has physical meaning. It's the initial length of the spring before we began adding weights. Our equation that models this spring's stretchy behavior is

$$
\operatorname{Length}(\mathrm{cm})=3.3 \frac{\mathrm{~cm}}{\mathrm{~N}} \times W \text { eight }(N)+12.9 \mathrm{~cm}
$$

Very nice! It sums up all the data in our graph plus everything outside it up to the elastic limit. A very tidy and efficient little package! If we kept adding weights until the spring began to distort, we'd find that the graph would start to bend toward the horizontal. So our equation only works within the spring's elastic range.

## By the way, who cares?

This might be a good time to discuss this burning question. Assuming that it's useful to be able to predict the length of a spring for example in designing cars, playground equipment, or whatever, what we've just done is really impressive.

We start with a real physical phenomenon which we then model by drawing a graph whose shape "behaves" just like the spring. From there we model the graph and hence the spring with an equation that "behaves" just like the spring and graph. The equation is now something that we can carry around in our head that is a perfect model of the behavior of the spring. This process is central to the study of physics and many other sciences. And it even allows you to answer questions like "how long is it going to be until we get to grandma's house." Without these models you could only say, "let's keep on driving until we get there and then we'll know."

### 2.1.3 - Direct Proportion

The equation we just found was somewhat determined by the way we set up the experiment. The spring's length was clearly changing so it was the obvious choice to compare to the weight. But if we had instead made measurements of the mass's position relative to its initial position, we would have been measuring the amount the spring stretched as weight was added. That seems closer to what a spring's behavior is all about.

It's a simple matter to rearrange the equation to fit that view of the spring's behavior. Subtracting the y-intercept, which is just the spring's initial length, from both sides gives us

$$
\text { Length }- \text { Initial Length }=3.3 \frac{\mathrm{~cm}}{\mathrm{~N}} \text { Weight }
$$

or

$$
\text { Stretch }=3.3 \frac{\mathrm{~cm}}{\mathrm{~N}} \text { Weight }
$$

Now we're comparing two quantities that are directly related to each other - the stretch and the weight, or force stretching the spring.

An extra column has been added below for you to record the stretch values by subtracting the spring's initial length from each length entry. Fill in those values and edit the y column in Grapher. Just click on the 12.8 and type 0.0 and then click <Enter>. The next stretch value will be ready to edit. You'll notice how the data point adjust each time you hit <Enter>. After the last entry you'll need to click in some other cell in the table to get the final point to adjust.

Relabel the graph title and axes to reflect the change. You should have something like Figure 10.

| Weight <br> $(\mathrm{N})$ | Length <br> $(\mathrm{cm})$ | Stretch <br> $(\mathrm{cm})$ |
| :---: | :---: | :---: |
| 0.0 | 12.8 | 0.0 |
| 0.5 | 14.8 | 2.0 |
| 1.0 | 16.2 |  |
| 1.5 | 17.7 |  |
| 2.0 | 19.9 |  |
| 2.5 | 21.2 |  |
| 3.0 | 22.5 |  |
| 3.5 | 24.4 |  |
| 4.0 | 26.6 |  |



Figure 10: Stretch vs. Weight

Again we have a linear fit to our data, but this time we can go further with our description. Since the stretch will be zero when the weight is zero, our new line will pass, within the range of error, through the origin of our graph. Since it has a constant slope and passes through the origin, the stretch equals a constant times the weight, $\mathbf{S}=\mathbf{k W}$. So doubling the weight will double the stretch, tripling the weight will triple the stretch, etc. This relation is called a direct proportion.

Note the contrast:

- In a linear relation the dependent variable changes by a fixed amount for a fixed change in the independent variable.
- A direct proportion is also a linear relationship, but in addition we see that two quantities are directly proportional if they vary in such a way that one of the quantities is a constant multiple of the other, or equivalently if they have a constant ratio.

Also worthy of note:

- A direct proportion implies a linear relation.
- A linear relation does not imply a direct proportion.


## Data Points vs. the Line of best fit and equations

Before moving on, pick one of the smaller weights in your data table and note the matching stretch. Now doubling the weight should result in twice the stretch. Check for that in your data table. Hmm. The stretch probably didn't exactly double! What gives?

No single data point determined the relationship we found. Rather a sort of averaging of the data was achieved by drawing our line of best fit and then finding its slope. Given the uncertainty in each measurement it wouldn't have been very useful to find an equation describing our actual data would it?

So when you're finished you have a data table and an equation. The equation is the best overall representation of your data. So don't look back at the data once you have your equation! Recall the similar statement about lines: "So once you create a line or curve, the data is no longer of any importance." If you count the moving fish in your fish pond three times and got 32,36 , and 37 , your best estimate of their number would be 35 , the average, not one of the specific counts. The line of best fit is a similar sort of average.

### 2.1.4 - Inverse Proportion

We found that in a direct proportion, multiplying the independent variable by a certain value will result in the dependent variable changing by the same factor. E.g., for a rectangle, Area $=$ length $\times$ width. Doubling the width will double the area. We'll also sometimes find the inverse of this relation where, for example, doubling one quantity will halve the other. Let's return to our pizza to find this inverse proportion. In that earlier analysis we looked at how the area of the pizza increased with diameter. This called for more and more pizza. (A cause we can all get behind!)

Times are hard now. Dough is in short supply. How can we continue to create the 20 inch pizzas we had earlier?
From a given amount of dough you can create a pizza of almost any size, but as the size (area) increases, something has to give. That something is thickness.

$$
\text { volume }=\text { area } \times \text { thickness }
$$

Let's assume our pizza dough comes in balls of diameter 4.6 in . Each one would then have a volume of

$$
\text { volume }=\frac{4}{3} \pi r^{3}=\frac{4}{3} \pi(2.3 \mathrm{in})^{3}=51 \mathrm{in}^{3}
$$

For any flat, (no rim stuffed with cheese!) circular pizza made with such a ball of dough,

$$
51 \text { in }^{3}=\text { area } \times \text { thickness }
$$

The graph of area vs. thickness in Figure 11 illustrates the nature of the inverse proportion. Since the product of the area and thickness is fixed, increasing one decreases the other by the same factor. That is, doubling the thickness halves the area, etc.

Two variables, x and y in this case are inversely proportional (or vary inversely) if their product is a constant, or equivalently one of the variables is directly proportional to the multiplicative inverse of the other.

This relation can be stated as

$$
k=x \times y \quad \rightarrow \quad x=\frac{k}{y} \quad \rightarrow \quad x \propto \frac{1}{y}
$$

- The product of x and y equals a constant

$$
k=x \times y
$$

- x is directly proportional to the inverse of $\mathrm{y} . \quad x=\frac{k}{y}$ or $x \propto \frac{1}{y}$

In this case the area of the pizza is inversely proportional to the thickness, or vice versa.

| Thickness <br> (in) | Area <br> $\left(\mathrm{in}^{2}\right)$ |
| :---: | :---: |
| 0.10 | 510 |
| 0.20 | 255 |
| 0.30 | 170 |
| 0.40 | 128 |
| 0.50 | 102 |
| 0.60 | 85 |
| 0.70 | 73 |
| 0.80 | 64 |
| 0.90 | 57 |
| 1.00 | 51 |



Figure 11: Area vs. Thickness

Just as with our initial pizza problem we can find the relation between the area and thickness by linearizing the data. If the area is inversely proportional to the thickness as we suspect, then the area is directly proportional to $1 /$ thickness. We need to create a new calculated column defined by $1 /$ thickness. But to keep the independent and dependent values in the proper order in our data table it's best to just replace the first column with our calculated $1 / \mathrm{T}$ values.
Click Auto to re-plot area vs. $1 / \mathrm{T}$. The new straight line indicates that area is directly proportional to $1 / \mathrm{T}$.
Turn on Linear Fit and select all the data points on the graph. Notice the familiar value of the slope.

| $1 / \mathrm{T}$ <br> $\left(\mathrm{in}^{-1}\right)$ | Area <br> $\left(\mathrm{in}^{2}\right)$ |
| :---: | :---: |
| 10.0 | 510 |
| 5.0 | 255 |
| 3.3 | 170 |
| 2.5 | 128 |
| 2.0 | 102 |
| 1.7 | 85 |
| 1.4 | 73 |
| 1.3 | 64 |
| 1.1 | 57 |
| 1.00 | 51 |



Figure 12: Area vs. 1/Thickness

$$
\begin{gathered}
\text { slope }=\frac{A}{1 / T}=A T=\text { Volume } \\
\text { Volume }=51.0=\text { Area } \times \text { Thickness }
\end{gathered}
$$

### 2.1.5 - Square or Quadratic Proportion

There are two different situations where we encounter quadratic proportions.

## a. Upward opening parabola <br> $y \propto x^{2}$ or $y=k x^{2}$ <br> b. Side opening parabola <br> $$
y^{2} \propto x \text { or } y^{2}=k x
$$

The dependent variable is directly proportional to the square of the independent variable.

The square of the dependent variable is directly proportional to the independent variable.

## a. Upward opening parabola

We've already seen the first one. By plotting Area vs. diameter squared we obtained a straight line with slope $.78 \mathrm{in}^{2}$ giving us the equation

$$
A=.78 i n^{2} d^{2}
$$

This is the equation of a parabola. More specifically it's an upward opening parabola. This is the nature of the graph of

$$
y \propto x^{2}
$$



Figure 13: Pizza Area vs. Diameter

## b. Side opening parabola

The other type of quadratic proportion is exhibited by waves traveling in a string such as those in a guitar. When a guitar string is plucked, waves travel quickly along the string and reflect back and forth between the fixed ends. The frequency of the sound produced is determined by the frequency of these round trips. The faster the waves travel the higher the frequency. The speed is adjusted by tightening and loosening the strings using the tuning pegs at the top of the guitar. Thus there is a relationship between the velocity of the waves and the tension in a string. Let's find it.
Enter the tension vs. speed data below into Grapher. You should see a different type of parabola in this case. It opens toward the side. (You should also be properly amazed at these speeds!)

| Tension <br> $(\mathrm{N})$ | Speed <br> $(\mathrm{km} / \mathrm{s})$ |  |
| :---: | :---: | :--- |
| 25 | .439 |  |
| 50 | .620 |  |
| 100 | .877 |  |
| 200 | 1.240 |  |
| 300 | 1.519 |  |
| 400 | 1.754 |  |
| 500 | 1.961 |  |
| 600 | 2.148 |  |
| 700 | 2.320 |  |
| 800 | 2.481 |  |
| 900 | 2.631 |  |



Figure 14: Speed vs. Tension

The tension values are increasing much faster than the speed values. Some function that would increase the dependent speed values more might work. There a many possibilities, but in this case, squaring the speed values will do the trick. Grapher can do this squaring for us without the need for the extra speed ${ }^{2}$ column shown below.

Add graph 2 if necessary. Click the Graphs button. Clicking the $\mathrm{Y}^{\wedge} 2$ button in the top row (2:) will square each speed value. The resulting graph is clearly linear. And since the $y$-intercept is zero, we have a direct proportion.

| Tension <br> $(\mathrm{N})$ | Speed <br> $(\mathrm{km} / \mathrm{s})$ | Speed $^{2}$ <br> $\left(\mathrm{~km}^{2} / \mathrm{s}^{2}\right)$ |
| :---: | :---: | :---: |
| 25 | .439 | .192 |
| 50 | .620 | .385 |
| 100 | .877 | .769 |
| 200 | 1.240 | 1.538 |
| 300 | 1.519 | 2.308 |
| 400 | 1.754 | 3.077 |
| 500 | 1.961 | 3.846 |
| 600 | 2.148 | 3.615 |
| 700 | 2.320 | 5.385 |
| 800 | 2.481 | 6.154 |
| 900 | 2.631 | 6.923 |



Figure 15: Speed vs. Tension

Putting our slope value into a direct proportion format $(y=k x)$ and noting that the $y$-value is $v^{2}$, gives us

$$
v^{2}=\left(.008\left(\frac{k m}{s}\right)^{2} / N\right) T
$$

This is again the equation of a parabola. This time we have a side opening parabola. This is the nature of the graph of

$$
y^{2} \propto x
$$

### 2.1.6 - A Summary of Mathematical Relationships and Related Graphs

In the lab we'll see relations 2-5 modeled. Relations 6-7 will be seen in class.
You'll want to refer back to this chart as you work with the data in your labs.

| Relation between variables | Graph | Equation |
| :---: | :---: | :---: |
| 1. No relation | Horizontal line | $\mathrm{y}=$ const. |
| 2. Linear relation with y-intercept | Straight, non-horizontal line not through origin | $y=m x+b$ <br> m : slope, b: y-intercept |
| 3. Direct proportion | Straight, non-horizontal line through origin | $\begin{aligned} & \mathrm{y}=\mathrm{mx} \\ & \mathrm{~m}: \text { slope } \end{aligned}$ |
| 4. Inverse proportion | Hyperbola | $\mathrm{y}=1 / \mathrm{x}$ |
| 5. Quadratic proportion | a. upward opening parabola <br> b. side opening parabola | $\begin{aligned} & \mathrm{y}=\mathrm{kx}^{2} \\ & \mathrm{y}^{2}=\mathrm{kx} \end{aligned}$ |
| 6. Inverse square proportion |  | $y=k / x^{2}$ |
| 7. Sinusoidal | (s) | $\begin{aligned} & \mathrm{y}=\mathrm{A} \sin (\mathrm{x}) \quad \text { (shown) } \\ & \mathrm{y}=\mathrm{A} \cos (\mathrm{x}) \\ & \mathrm{y}=\mathrm{A} \tan (\mathrm{x}) \end{aligned}$ |

## 3. - Capturing Graphs and Sketches for Uploading in WebAssign Or Printing

Most of the labs include a Screenshot tool which is used for capturing and saving images of parts of the apparatus, graphs, and figures. In some labs you'll also create sketches right from the apparatus. You'll upload these images into WebAssign and sometimes print them out to paste into your lab reports. The method works the same for sketches and graphs. In both cases you'll take a screenshot of some part of the lab apparatus or Grapher. We'll first look at the Screenshot tool. Then we'll look at a typical example of creating sketches.

### 3.1 Uploading graphs and sketches for grading using the "Screenshot" tool.

A screenshot can be taken from Grapher or any lab apparatus that includes the Screenshot tool icon. There are two steps to uploading a screenshot.

- Take a Screenshot and save it locally to your hard drive, a thumb drive, or similar.
- Upload that local file from within the lab you're working on in WebAssign.

Let's try it with whatever graph you have on your screen. If you don't have Grapher open, do that now.

1. Take a Screenshot and save it locally to your hard drive, a thumb drive, or similar. Here's how.
a. Start the Screenshot process by clicking the Screenshot icon which is a small camera image.
b. Your mouse pointer will change to a cross-hair.

Without clicking, move it to the top-left corner of the area you want to capture. Click and drag to the bottom right corner of the capture area and release. (You can actually use any two diagonally opposite corners.)
c. When you release the mouse button a file requester will pop up. Navigate to wherever you want to save your captured image and change the default name to the name suggested in the lab. Be sure to include the ".png" at the end. Use "test.png" or similar for this example.


Figure 16: File Requester
Note: When you're asked to create a screenshot you'll also be provided a filename to use so that you can easily keep track of these files. For example, in one lab you'll be asked to "save it as "Vel_IB1a.png" and then upload it." You'll save the file locally with that name. You can actually create these image files anytime you like and upload them to WebAssign as needed.
2. Upload the local file that you've created and saved from within the lab you're working on in WebAssign.
a. At a point in a lab where you've been told to create a graph or sketch you'll be asked to upload it. WebAssign will include a file requester that looks something like this:


## WebAssign Upload Tool

b. You'll click "Browse" and search on your local device for the correct file. Click on the filename and then click Open. The file requester will close and you'll see the file name beside the Browse button. You're done.
c. If you need to replace the file just repeat the process using the same name.

Also, if you forget the .png extension, you can edit the local file name to include it and then do the upload.

### 3.2 Creating Sketches

In the study of buoyancy you'll learn how the apparent weight of an object decreases as it's lowered into a fluid such as water. The boxed text (\#1 and \#3) and figures below show how we explore this behavior. In Figure 17a, a cork is hanging from a spring scale used to weigh the cork under various conditions. Initially the scale just registers the actual weight of the cork. The instructions below (\#1) ask you to take Screenshots of figures 17a, 17c, and 17d to record your results.
\#1. In the questions that follow you'll submit your responses for Figure 17a, 17c, and 17d by uploading Screenshots 0 of each. For 17a and 17c just capture an image of the scale. For 17d, capture an image of the digital scale reading.


Figure 19a below shows a typical screenshot showing that the scale reads about .7 N . What do you think the scale will read in Figure 17c? You'll find out when you do the lab.

The next step (\#3) asks you to create an FBD, a free-body diagram, for the cork in the conditions shown in Figure 17c above. Notice that the string is slack which indicates that the cork is floating. There are two forces acting on the cork the force of gravity, that is, it's weight, W, and the buoyant force, Fb .
\#3. Draw FBD 19e to the left of the floating cork provided. Also use the Sketch tool $\wedge$ to create a screen-sketch by adding and labeling a pair of vector arrows to the left of the floating cork and tank in the Buoyancy Apparatus. Take a Screenshot of the FBD and the top of the tank and save it as "Buoy_FBD19e.png" and upload it.

To create this sketch you'll use the Sketch tool shown in Figure 18. The correct FBD will display two vector arrows drawn to scale. Each needs to be labeled as indicated in \#3. When you click on the arrow tool, a horizontal blue arrow appears. (Fig 19b.) Clicking on the head of the arrow (19c) allows you to rotate it and adjust its length. You can then click on its body (19d) and drag it to where it needs to be (next to the floating cork), and add a label using the text gadget (19e). The circle is just used to drag the text around.


Figure 18: the Sketch Tool

There are other types of sketch tools but they all work similarly. Further instructions are provided within the labs.

