## 22-1 Maxwell's Equations

In the 19th century, many scientists were making important contributions to our understanding of electricity, magnetism, and optics. For instance, the Danish scientist Hans Christian Ørsted and the French physicist André-Marie Ampère demonstrated that electricity and magnetism were related and could be considered part of one field, electromagnetism. A number of other physicists, including England's Thomas Young and France's Augustin-Jean Fresnel, showed how light behaved as a wave. For the most part, however, electromagnetism and optics were viewed as separate phenomena.

James Clerk Maxwell was a Scottish physicist who lived from 1831 – 1879. Maxwell advanced physics in a number of ways, but his crowning achievement was the manner in which he showed how electricity, magnetism, and optics are inextricably linked. Maxwell did this, in part, by writing out four deceptively simple equations.

Physicists love simplicity and symmetry. To a physicist, it is hard to beat the beauty of Maxwell's equations, shown in Figure 22.1. To us, they might appear to be somewhat imposing, because they require a knowledge of calculus to fully comprehend them. These four equations are immensely powerful, however. Together, they hold the key to understanding much of what is covered in this book in Chapters 16 – 20, as well as Chapters 22 and 25. Seven chapters boiled down to four equations. Think how much work we would have saved if we had just started with Maxwell's equations instead, assuming we understood all their implications immediately.

Equation 1: $\int \vec{E} \bullet d\vec{A} = \dfrac{Q}{\varepsilon_0}$  Equation 3: $\int \vec{E} \bullet d\vec{l} = -\dfrac{d\Phi_B}{dt}$

Equation 2: $\int \vec{B} \bullet d\vec{A} = 0$  Equation 4: $\int \vec{B} \bullet d\vec{l} = \mu_0 I_{enclosed} \; \boxed{+ \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt}}$ — term added by Maxwell

**Figure 22.1**: Maxwell's equations.

**Understanding Maxwell's equations.**

Equation 1 is known as Gauss' Law for electric fields. It tells us that electric fields are produced by charges. From this equation, Coulomb's Law can be derived. Note that the constant $\varepsilon_0$ in Equation 1, the permittivity of free space, is inversely related to $k$, the constant in Coulomb's Law: $k = 1/(4\pi \varepsilon_0)$.

Equation 2 tells us that magnetic field lines are continuous loops.

Equation 3 is Faraday's Law in disguise, telling us that electric fields can be generated by a magnetic flux that changes with time.

Consider Equation 4. Setting the left side equal to the first term on the right-hand side, we have an equation known as Ampère's law, which tells us that magnetic fields are produced by currents. Everything up to this point was known before Maxwell. However, when Maxwell examined the equations (the first three, plus Equation 4 with only the first term on the right) he noticed that there was a distinct lack of symmetry. The equations told us that there were two ways to create electric fields (from charges, or from changing magnetic flux), but they only had one way to create magnetic fields (from currents). One of Maxwell's major contributions, then, was to bring in the second term on the right in Equation 4. This gave a second way to generate magnetic fields, by electric flux that changed with time, making the equations much more symmetric.

Maxwell did not stop there. He then asked the interesting question, what do the equations predict if there are no charges and no currents? Equations 3 and 4 say that, even in the absence of charges and currents, electric and magnetic fields can be produced by changing magnetic and electric flux, respectively. Furthermore, Maxwell found that when he solved the equations, the solutions for the electric and magnetic fields had the form $E(x,t) = E_0 \cos(\omega t - kx)$ and $B(x,t) = B_0 \cos(\omega t - kx)$. We recognize these, based on what we learned in Chapter 21, as the equations for traveling waves.

Finally, Maxwell derived an equation for the speed of the traveling electric and magnetic waves,

$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = \sqrt{\frac{1}{(4\pi \times 10^{-7} \text{ Tm/A})(8.85 \times 10^{-12} \text{ C}^2/(\text{Nm}^2))}} = 3.00 \times 10^8 \text{ m/s} .$$

(**Equation 22.1**: Maxwell's derivation of the speed of light)

Maxwell recognized that this speed was very close to what the French physicists Hippolyte Fizeau and Léon Foucault had measured for the speed of light in 1849. Thus, Maxwell proposed (in 1873) that light consists of oscillating electric and magnetic fields in what is known as an electromagnetic (EM) wave. It was 15 years later, in 1888, that Heinrich Hertz, from Germany, demonstrated the production and detection of such waves, proving that Maxwell was correct.

Despite Hertz's experimental success, he rather famously stated that he saw no application for electromagnetic waves. Only 120 or so years later, the world has been transformed by our use of electromagnetic waves, from the mobile phones (and other communication devices) that almost all of us carry around, to the radio, television, and wireless computer signals that provide us with entertainment and information, and to x-rays used in medical imaging.

Throughout the 19th century, evidence for light behaving as a wave piled up very convincingly. However, all previous known types of waves required a medium through which to travel. Scientists spent considerable effort searching for evidence for the medium that light traveled through, the so-called luminiferous aether, which was thought to fill space. Such an aether could, for instance, explain how light could travel through space from the Sun to Earth. An elegant experiment in 1887, by the American physicists Albert Michelson and Edward Morley, showed no evidence of such a medium, and marked the death knell for the aether idea. Our modern understanding is that light, or any electromagnet wave, does not require a medium through which to travel. This view is consistent with Maxwell's equations. Electromagnetic waves consist of oscillating electric and magnetic fields, and by Maxwell's equations such time-varying fields produce oscillating magnetic and electric fields, respectively. Hence, an electromagnetic wave can be thought of as self-sustaining, with no medium required.

Finally, note how Equation 22.1 reinforces the idea that light, electricity, and magnetism are all linked. The equation brings together three constants, one associated with light ($c$, the speed of light), one associated with magnetism ($\mu_0$, which appears in equations for magnetic field), and one associated with electricity ($\varepsilon_0$, which appears in equations for electric field).

**Related End-of-Chapter Exercises: 36, 37, 38.**


***Essential Question 22.1*** In Chapter 19, we used the equation $E/B = v$ when we discussed the velocity selector. Use this to help show how the units in Equation 22.1 work out.