Answer to Essential Question 13.3: First, the two methods are completely equivalent, and they should give the same answer. There is a difference, however, in how signs are handled for the ΔT 's. In the $\sum Q = 0$ method, each ΔT is specified as a final temperature minus an initial temperature, so a ΔT can be positive (if the temperature increases) or negative (if the temperature decreases). In the heat lost vs. heat gained method, all the ΔT 's are taken to be positive. In the case of the lead ball and water, the analysis would proceed like this:

$Q_{gained} = Q_{lost}$ so $m_{H_2O}\, c_{H_2O} \left|\Delta T_{H_2O}\right| = m_{Pb}\, c_{Pb} \left|\Delta T_{Pb}\right|$.

Using the absolute values of the ΔT 's ensures that every term in the equation is positive.

13-4 Latent Heat

Let's expand our knowledge of thermal equilibrium problems by learning to handle phase changes. In general, a change of phase is associated with a relatively large amount of energy, and occurs at a particular temperature. To melt ice, for instance, first heat is added to raise the ice's temperature to 0°C, the melting point. Adding more heat breaks bonds and gradually transforms the solid water into liquid water. The transformation takes place at a constant temperature of 0°C.

Because there is no change in temperature associated with a phase change, the equation we use has a different form than the $Q = mc\Delta T$ equation we use for the heat required to change a substance's temperature. The amount of heat associated with a phase change is given by:

$Q = mL_f$, (Equation 13.10: **Heat for a liquid-solid phase change**)

where L_f is known as the **latent heat of fusion,** or,

$Q = mL_v$, (Equation 13.11: **Heat for a gas-liquid phase change**)

where L_v is known as the **latent heat of vaporization**.

As shown in Table 13.3, latent heat values depend on the material. To melt or vaporize a substance requires that heat is added, while heat must be removed from a substance to solidify or condense it. Table 13.4 shows melting points and latent heats of fusion of some common metals.

Material	Melting point	Latent heat of fusion (kJ/kg)	Boiling point	Latent heat of vaporization (kJ/kg)
Water	0°C	335	100°C	2272
Nitrogen	-210°C	25.7	-196°C	200
Oxygen	-219°C	13.9	-183°C	213
Ethyl alcohol	-114°C	108	-78.3°C	855

Table 13.3: A table of melting points, boiling points, and latent heats for various materials.

Material	Melting point	Latent heat of fusion (kJ/kg)
Aluminum	933.5K	396
Copper	1356.6K	209
Gold	1337.58K	64
Iron	1808K	250
Lead	600.65K	23

Table 13.4: A table of melting points and latent heats of fusion for various metals.

EXPLORATION 13.4 – Temperature vs. time

100 grams of ice at –20°C is put in a pot on a burner on the stove. The burner transfers energy to the water at a rate of 400 W. The ice melts, and eventually all the water boils away.

Step 1 – ***How many different heat terms do we need to keep track of in this process? Describe them in words.*** There are four heat terms we need to keep track of. These include:

1. The heat needed to increase the temperature of the ice to the melting point.
2. The heat needed to melt the ice at 0°C.
3. The heat needed to raise the liquid water to the boiling point.
4. The heat needed to boil the water at 100°C.

Step 2 – ***Plot a graph of the temperature of the water as a function of time, starting at t = 0 when the temperature is at –20°C.*** To draw the graph it's helpful to find the time each of the four steps takes in the process. Because energy is power multiplied by time, the times can be found by dividing the heat in each case by the power.

Raising the temperature of the ice takes $t_1 = \frac{mc\Delta T}{P} = \frac{(0.100\,\text{kg})(2060\,\text{J/kg°C})(+20\text{°C})}{400\,\text{J/s}} = 10.3\,\text{s}$.

Melting the ice takes $t_2 = \frac{mL_f}{P} = \frac{(0.100\,\text{kg})(3.33\times10^5\,\text{J/kg})}{400\,\text{J/s}} = 83.25\,\text{s}$.

Raising the water by 100°C takes $t_3 = \frac{mc\Delta T}{P} = \frac{(0.100\,\text{kg})(4186\,\text{J/kg°C})(+100\text{°C})}{400\,\text{J/s}} = 105\,\text{s}$.

Boiling the water takes $t_4 = \frac{mL_V}{P} = \frac{(0.100\,\text{kg})(2.256\times10^6\,\text{J/kg})}{400\,\text{J/s}} = 564\,\text{s}$.

The total time required to vaporize all the water is t = 762 s, which is equivalent to 12.7 minutes. The water spends most of the time changing phase, particularly while it is vaporizing. This tells us that it takes a lot of energy to change phase. We can also see the large fraction of time spent changing phase on the graph of the temperature as a function of time in Figure 13.4.

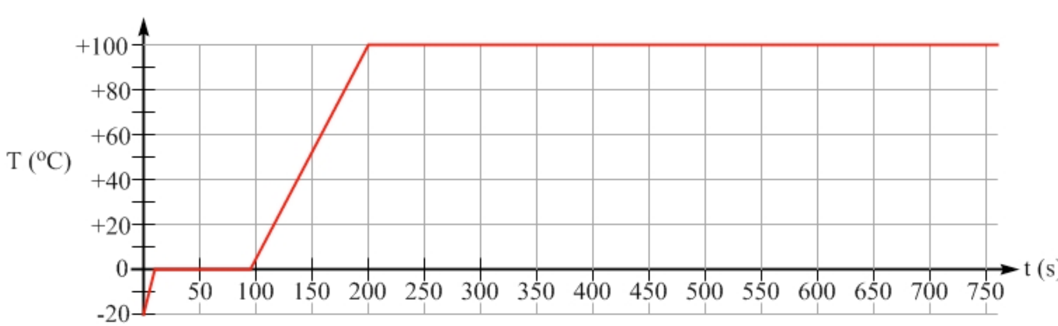

Figure 13.4: A plot of the temperature as a function of time (in seconds) for a sample of water that starts as a solid at –20°C and ends up boiling away completely. Note the large fraction of the time the water spends changing phase, in the two constant-temperature sections of the graph.

Key idea regarding phase changes: Changes of phase are generally associated with a relatively large amount of energy. **Related End-of-Chapter Exercises: 4, 31, 32.**

Essential Question 13.4: Return to the graph in Figure 13.4. Why is the slope of the graph constant when the sample's temperature is changing? What is the slope of the graph equal to?